8.2 L'Hospital's Rule

- I can recognize limits in indeterminate form

-I can evaluate limits applying L'Hospital's rule

Mar 10-11:29 AM

Evaluate the limit using substitution:

indeterminate form

$$\lim_{x \to 0} \frac{\sin x}{x} = \frac{\sin(0)}{0} = \frac{0}{0}$$

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} = \frac{\infty}{\infty}$$

Nov 19-1:48 PM

Evaluate the following limits

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

Mar 10-11:30 AM

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

so long as the limit is finite, $+\infty$, or $-\infty$. Similar results hold for $x \to \infty$ and $x \to -\infty$.

THEOREM 2 (l'Hopital's Rule for infinity over infinity): Assume that functions f and g are differentiable for all x larger than some fixed number. If $\lim\limits_{x \to a} f(x) = \infty$ and $\lim\limits_{x \to a} g(x) = \infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

so long as the limit is finite, $+\infty$, or $-\infty$. Similar results hold for $x \to \infty$ and $x \to -\infty$.

In both forms of l'Hopital's Rule it should be noted that you are required to differentiate (separately) the numerator and denominator of the ratio if either of the indeterminate forms $\frac{"0"}{0}$ or $\frac{"\infty"}{\infty}$ arises in the computation of a limit. Do not confuse l'Hopital's Rule with the Quotient Rule for derivatives. Here is a simple illustration of Theorem 1.

Nov 19-2:18 PM

Use L'Hospital's rule evaluate the following and support graphically

indeterminate form

$$\lim_{x\to 0}\frac{\sin x}{x} = \frac{0}{0} = \lim_{x\to 0}\frac{\cos x}{1} = \frac{\cos(0)}{1} = \frac{1}{1} = 1$$

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4} = \frac{0}{0}$$

$$\lim_{x\to 2}\frac{2x-3}{2x} = \frac{2(2)-3}{2(2)} = \frac{1}{4}$$

$$\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty}\frac{\sqrt{x}}{x} = \lim_{x\to\infty}\frac{1}{\sqrt{x}} = 0$$

$$\frac{x^{\frac{1}{2}}}{x^1}$$

$$2x^{\frac{1}{2}}$$

$$x^{-\frac{1}{2}}$$

$$\frac{1}{\sqrt{x}}$$

Mar 10-11:39 AM

Apply L'Hospital's rule to evaluate the following limits

1. $\lim_{x\to 1}\frac{\ln x}{x-1}$

2. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

Mar 10-11:42 AM

Apply L'Hospital's rule to evaluate the following limits

3. $\lim\limits_{x\to\pi} \dfrac{\pi - x}{\sin x}$ $= \dfrac{\pi - \pi}{\sin \pi} = \dfrac{0}{0} = \lim\limits_{x\to\pi} \dfrac{-1}{\cos x}$

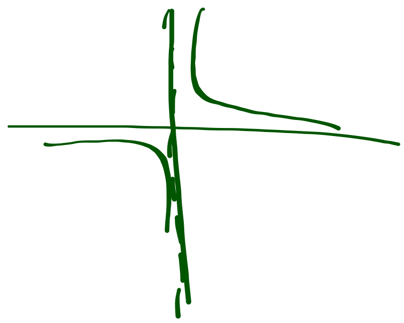
$= \dfrac{-1}{\cos \pi} = \dfrac{-1}{-1} = 1$

Mar 10-11:44 AM

Apply L'Hospital's rule to evaluate the following limits

6. $\lim\limits_{x\to 0} \dfrac{\overset{4x}{\cancel{1-\cos x}}}{x^2} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{4}{2x} = \dfrac{4}{0}$ undefined

$= DNE$



Mar 10-11:48 AM