Quiz 7.2

1 (2 point) Simplify the following expression.

$$\left(25^{\frac{3}{4}} \bullet 4^{-\frac{3}{4}}\right)^2$$

2. (2 point) Distribute and simplify.

$$x^{\frac{1}{2}}\left(x^{\frac{3}{2}} - 2\right)$$

3. (2 point) Simplify;

$$\sqrt[4]{x^2} - \frac{\sqrt[4]{x^6}}{x}$$

7.3 Simplifying Radical Expressions

Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $n \geq 2$ is an integer, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

We can prove this using rational exponents.

$$(a)^{\frac{1}{n}} \cdot (b)^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$$

Multiply

$$\sqrt{5} \cdot \sqrt{3} = \sqrt{15}$$

$$\sqrt[3]{2} \cdot \sqrt[3]{13}$$

$$\sqrt[3]{26}$$

Multiply

$$\sqrt[5]{6c} \cdot \sqrt[5]{7c^2}$$

$$\sqrt{x-3} \cdot \sqrt{x+3}$$

$$\sqrt{(x-3)(x+3)}$$

$$\sqrt{(x^2-9)} = \cancel{x-3}$$

You try

$\sqrt{11}\cdot\sqrt{7}$ $\sqrt[4]{6}\cdot\sqrt[4]{7}$

$\sqrt{x-5}\cdot\sqrt{x+5}$ $\sqrt[7]{5p}\cdot\sqrt[7]{4p^3}$

Simplify

$\sqrt{18} = \sqrt{9}\cdot\sqrt{2}$

9 $\sqrt{2}$

3 3

$\sqrt{3^2}$

$= 3\sqrt{2}$

$3\sqrt{2}$

Simplify

$5\sqrt[3]{24}$

24

6 4

3 2 2 2

$\sqrt[3]{8}\cdot\sqrt[3]{3}$

$10\sqrt[3]{3}$

(remember $\sqrt{x^2}=|x|$)

$\sqrt{128x^2}$

$\sqrt{64}\cdot\sqrt{2}\cdot\sqrt{x^2}$

$8|x|\sqrt{2}$

$\sqrt[4]{20}$

You try

$\sqrt{48}$ $4\sqrt[3]{54}$

$\sqrt{200a^2}$ $\sqrt[4]{40}$

Simplify (There are 2 different ways)

$$\frac{4-\sqrt{20}}{2} = \frac{4-2\sqrt{5}}{2} = \frac{4}{2} - \frac{2\sqrt{5}}{2}$$

$$= 2-\sqrt{5}$$

You Try

$$\frac{6-\sqrt{45}}{3}$$

Scratch

$$\sqrt{4}\cdot\sqrt{4}\cdot\sqrt{2}$$

$$\frac{-2+\sqrt{32}}{4}$$

$$\frac{-2+4\sqrt{2}}{4} = -\frac{1}{2}+\sqrt{2}$$

Remember that

$\sqrt[n]{a^n} = a$ if $n \geq 3$ is odd

$\sqrt[n]{a^n} = |a|$ if $n \geq 2$ is even

For example

$\sqrt{x^2} = |x|$ $\sqrt[3]{x^3} = x$ $\sqrt[4]{x^4} = |x|$ and so on

But to make our life easier some instructions will say "Assume all variables are greater then or equal to zero." In which case:

$\sqrt{x^2} = x$ $\sqrt[3]{x^3} = x$ $\sqrt[4]{x^4} = x$ and so on

SO READ YOUR INSTRUCTIONS!!!

Reduce Assuming all variables are greater then or equal to zero.

(You can either do these using rational exponents or not.)

$$\sqrt{x^6} = \sqrt{x^2}\cdot\sqrt{x^2}\cdot\sqrt{x^2}$$

$$= x^3$$

$$\sqrt[3]{x^{12}} \quad \sqrt[3]{x^3}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{x^3}$$

$$= x^4$$

Reduce Assuming all variables are greater then or equal to zero.

$$\sqrt{20x^{10}}$$

$2x^5\sqrt{5}$

You try

$$\sqrt{75a^6}$$

Simplify Assuming all variables are greater then or equal to zero.

$$\sqrt{80a^3} \qquad \sqrt[3]{27m^4n^{14}}$$

You Try

$$\sqrt[3]{128x^6y^{10}}$$

$2 \cdot 2 \cdot 2 \cdot 2$

$$\sqrt[4]{16a^5b^{11}}$$

$2ab^2\sqrt[4]{ab^3}$

Multiply and Simplify Assuming all variables are greater then or equal to zero.

$$\sqrt{3}\cdot\sqrt{15}$$

$= \sqrt{45}$

$= \sqrt{9}\cdot\sqrt{5}$

$3\sqrt{5}$

$$3\sqrt[3]{4x}\cdot\sqrt[3]{2x^4}$$

$3\sqrt[3]{8x^5}$

$6x\sqrt[3]{x^2}$

$$\sqrt[4]{27a^2b^5}\cdot\sqrt[4]{6a^3b^6}$$

Multiply and Simplify Assuming all variables are greater then or equal to zero.

$$\sqrt{3}\cdot\sqrt{15}$$

$$3\sqrt[3]{4x}\cdot\sqrt[3]{2x^4}$$

$$\sqrt[4]{27a^2b^5}\cdot\sqrt[4]{6a^3b^6}$$

You try

$$\sqrt{6}\cdot\sqrt{8}$$

$$4\sqrt[3]{8a^2b^5}\cdot\sqrt[3]{6a^2b^4}$$

Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, $n \geq 2$ is an integer, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

We can prove this using rational exponents.

Simplify Assuming all variables are greater then or equal to zero.

$\sqrt{\frac{18}{25}}$ $\sqrt[3]{\frac{6z^3}{125}}$ $\sqrt[4]{\frac{10a^2}{81b^4}}$

$\frac{\sqrt{18}}{\sqrt{25}} = \frac{3\sqrt{2}}{5}$

You try

$\sqrt{\frac{13}{49}}$ $\sqrt[3]{\frac{27p^3}{8}}$ $\sqrt[4]{\frac{3q^4}{16}}$

Simplify Assuming all variables are greater then or equal to zero.

$\frac{\sqrt{24a^3}}{\sqrt{6a}} = \sqrt{\frac{24a^3}{6a}}$ $\frac{-2\sqrt[3]{54a}}{\sqrt[3]{2a^4}}$

$= \sqrt{4a^2}$

$= 2a$

$\frac{\sqrt[3]{-375x^2y}}{\sqrt[3]{3x^{-1}y^7}}$

You try

$\frac{\sqrt{12a^5}}{\sqrt{3a}}$ $\frac{\sqrt[3]{-24x^2}}{\sqrt[3]{3x^{-1}}}$

$\frac{\sqrt[3]{250a^5b^{-2}}}{\sqrt[3]{2ab}}$

Multiply and Simplify

$$\sqrt[4]{8}\cdot\sqrt[3]{5}$$

$= (8)^{\frac{1}{4}}\cdot(5)^{\frac{1}{3}}$

$= (8)^{\frac{3}{12}}\cdot(5)^{\frac{4}{12}} \rightarrow \left(8^3\cdot 5^4\right)^{\frac{1}{12}}$

$= \sqrt[12]{8^3}\cdot\sqrt[12]{5^4}$ $\quad (320000)^{\frac{1}{12}}$

$= \sqrt[12]{320000}$ $\quad = \sqrt[12]{320000}$

You try

$$\sqrt[4]{5}\cdot\sqrt[3]{3}$$

$= (5)^{\frac{1}{4}}\,(3)^{\frac{1}{3}}$

$= (5)^{\frac{3}{12}}\cdot(3)^{\frac{4}{12}}$

$= \left(5^3\cdot 3^4\right)^{\frac{1}{12}}$

$$\sqrt{10}\cdot\sqrt[3]{12}$$

$-10, -9.99999\ldots, -9.999_8$

$\sqrt[4]{16} = (2)$

$\sqrt{15}$