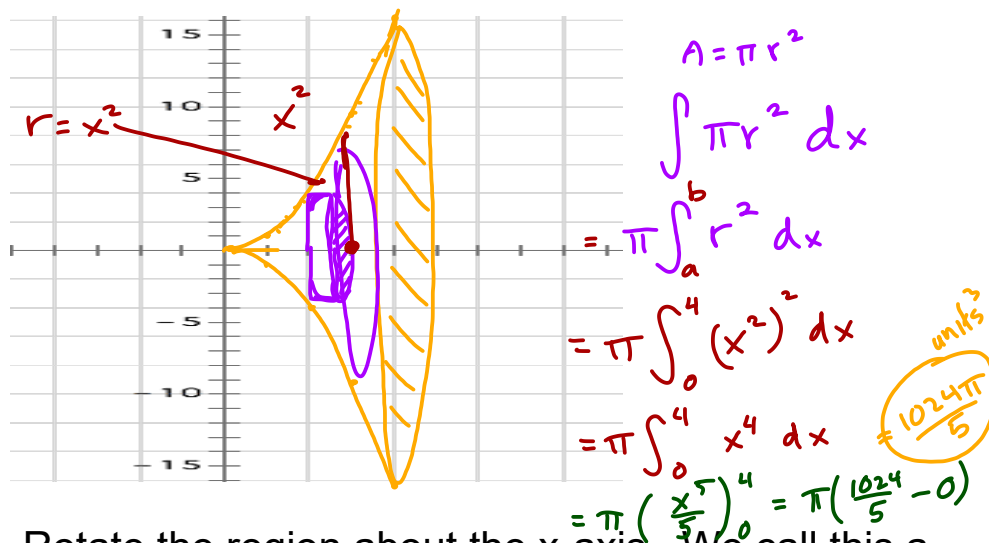


## 7.3.1 Solids of Revolution

Consider the region bound by  $f(x) = x^2$  and the x-axis over  $[0,4]$



Rotate the region about the x-axis. We call this a solid of revolution. How could we find the volume of the solid?

Feb 27-5:42 PM

Find the volume of the solid generated by rotating the enclosed region about the x-axis.

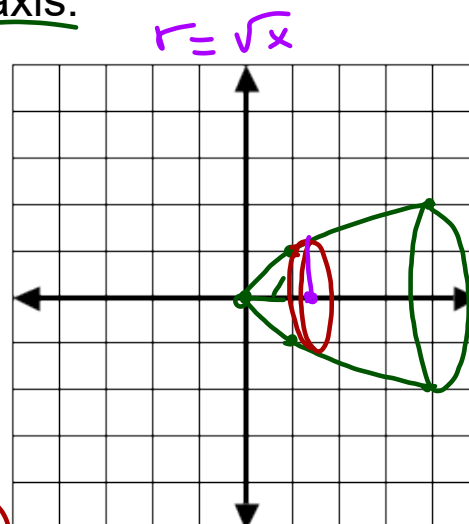
1.  $y = \sqrt{x}$ ,  $x = 4$ ,  $y = 0$

$$= \pi \int_0^4 (\sqrt{x})^2 dx$$

$$= \pi \int_0^4 x dx$$

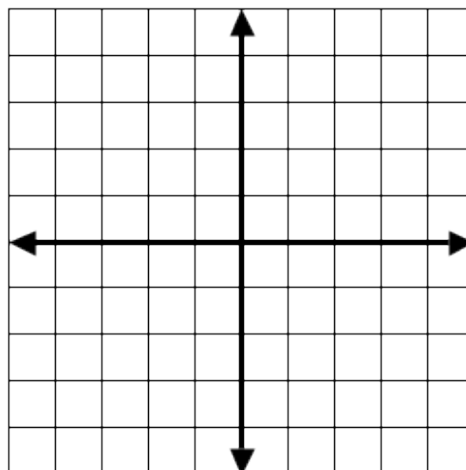
$$= \pi \left( \frac{x^2}{2} \right)_0^4$$

$$= 8\pi - 0\pi = 8\pi \text{ units}^2$$



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5.  $y = \sin x, y = 0$   $[0, \pi]$



Feb 27-6:05 PM

Washer Method

6.  $y = x^2, y = x^3$



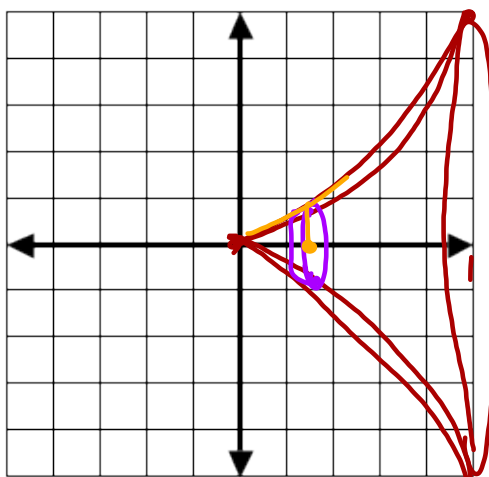
$$\pi R^2 - \pi r^2$$

$$\pi (R^2 - r^2)$$

$$= \pi \int_0^1 R^2 - r^2 dx$$

$$= \pi \int_0^1 (x^2)^2 - (x^3)^2 dx$$

$$= \pi \int_0^1 x^4 - x^6 dx$$



$$= \pi \left( \frac{x^5}{5} - \frac{x^7}{7} \right)_0^1$$

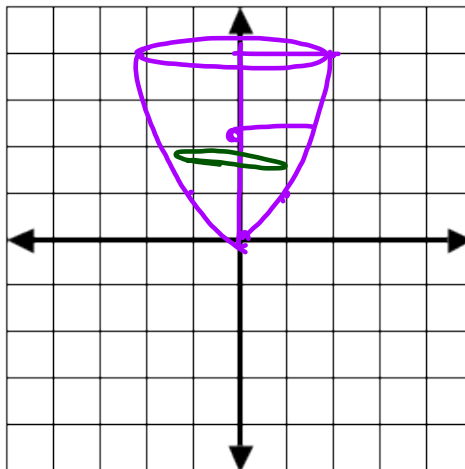
$$= \pi \left( \frac{1}{5} - \frac{1}{7} \right)$$

$$= \pi \left( \frac{2}{35} \right) = \frac{2\pi}{35} \text{ units}^3$$

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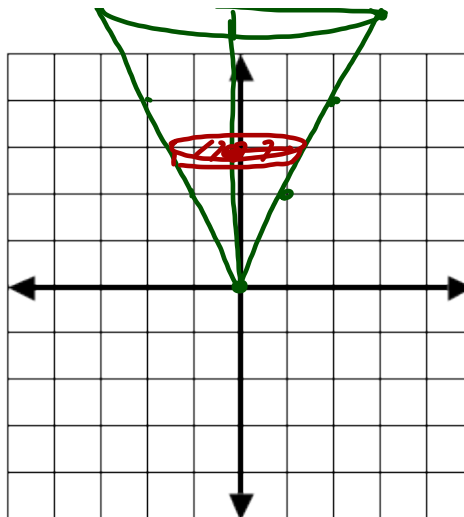
Find the volume of the solid generated by rotating the enclosed region about the y-axis.

$$\begin{aligned}
 8. \sqrt{y} &= \sqrt{x^2}, \quad x=0, \quad y=4 \\
 x &= \sqrt{y} \\
 &= \pi \int_0^4 (\sqrt{y})^2 dy \\
 &= \pi \int_0^4 y dy \\
 &= \pi \left( \frac{y^2}{2} \right)_0^4 \\
 &= \pi (8 - 0) \\
 &= 8\pi \text{ units}^3
 \end{aligned}$$



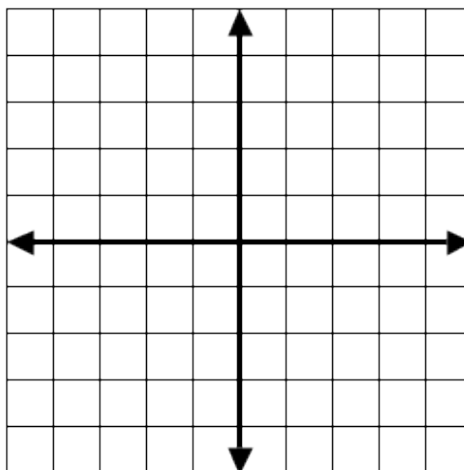
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$$\begin{aligned}
 9. y &= 2x, \quad x=0, \quad y=6 \\
 x &= \frac{1}{2}y \\
 &= \pi \int_0^6 \left( \frac{1}{2}y \right)^2 dy \\
 &= \frac{\pi}{4} \int_0^6 y^2 dy \\
 &= \frac{\pi}{4} \left( \frac{y^3}{3} \right)_0^6 \\
 &= \frac{\pi}{4} (54 - 0) \\
 &= 18\pi
 \end{aligned}$$



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10.  $y = x^2$ ,  $x = 4$ ,  $y = 0$



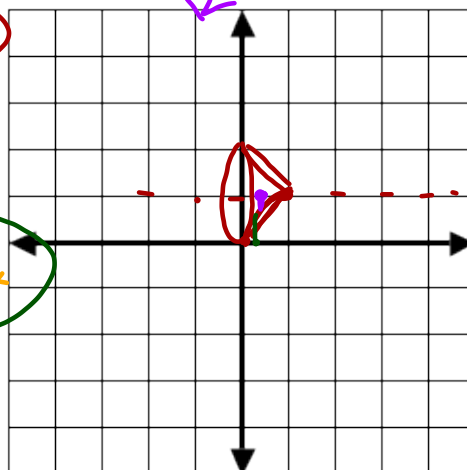
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Find the volume of the solid generated by rotating the enclosed region about the given axis.

11.  $y = x^3$ ,  $y = \sqrt{x}$  about  $y = 1$

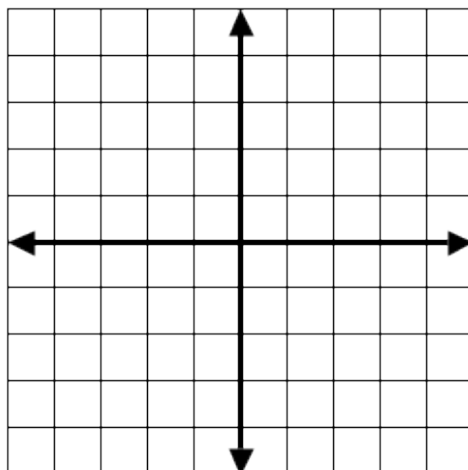
$$\pi \int_0^1 R^2 - r^2 dx$$

$$\pi \int_0^1 (1-x^3)^2 - (1-\sqrt{x})^2 dx$$



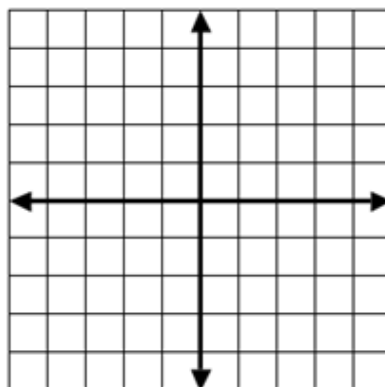
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12.  $y = x^3$ ,  $y = \sqrt{x}$  about  $y = -3$



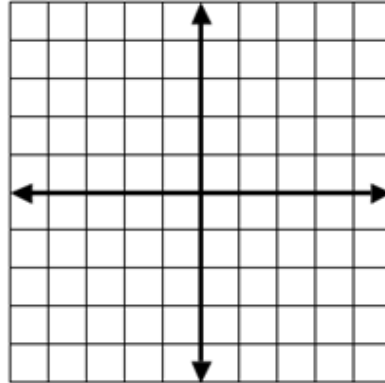
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3.  $y = x^2$ ,  $x = 4$ ,  $y = 0$ , about  $x = 0$



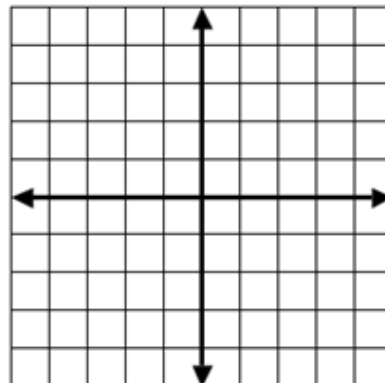
Mar 21-9:19 PM

5.  $y = x^2$ ,  $y = 0$ ,  $x = 2$ , about  $y = -2$



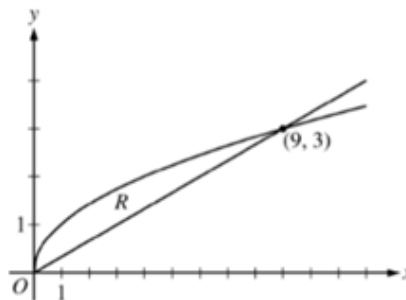
Mar 21-9:22 PM

7.  $y = x^2$ ,  $y = 0$ ,  $x = 2$ , about  $x = -1$



Mar 21-9:25 PM

Let  $R$  be the region in the first quadrant enclosed by the graphs of  $g(x) = \sqrt{x}$   
 $h(x) = \frac{x}{3}$



a) Find the area of region  $R$ .

b) Write, but do not evaluate, an expression that gives the volume of the solid generated when  $R$  is revolved about the horizontal line  $y=4$ .

Feb 7-9:35 AM