6.4 Initial Value and Exponential Growth and Decay

Find the solution to the initial value problem.

1. $\frac{dy}{dx} = \frac{2x}{y}$ if $y = 4$ when $x = 3$

1) Sep Vars
2) Integrate
3) Use init. vals to find c
4) Subst in c and solve for y

$\int y\,dy = \int 2x\,dx$

$\frac{y^2}{2} = x^2 + c$

$\frac{4^2}{2} = 3^2 + c$

$8 = 9 + c$

$c = -1$

$\frac{y^2}{2} = x^2 - 1$

$y^2 = 2x^2 - 2$

$y = \pm\sqrt{2x^2 - 2}$

$y = \sqrt{2x^2 - 2}$

Feb 3-1:41 PM

2. $\frac{dy}{\cancel{y}\,dx} = 4x\cancel{y}$ ~~if~~ $y = e^4$ when $x = 1$

$\frac{dy}{y} = 4x\,dx$

$\int \frac{1}{y}\,dy = \int 4x\,dx$

$\ln|y| = 2x^2 + c$

$\ln e^4 = 2(1)^2 + c$

$4 = 2 + c$

$c = 2$

$e^{\ln|y|} = e^{(2x^2 + 2)}$

$y = e^{2x^2 + 2}$

or $y = e^{2x^2} \cdot e^2$

$x^2 \cdot x^3 = x^5$

Feb 3-5:01 PM

3. $\frac{dy}{dx} = e^{x+y}$ *if* $y = 4$ *when* $x = 0$

$$\frac{dy}{dx} = e^x \cdot e^y$$

$$\frac{1}{e^y}\,dy = e^x\,dx$$

$$\int e^{-y}\,dy = \int e^x\,dx$$

$$-e^{-y} = e^x + c$$

$$-e^{-4} = e^0 + c$$

$$-e^{-4} = 1 + c$$

$$c = -e^{-4} - 1$$

$$-e^{-y} = e^x - e^{-4} - 1$$

$$e^{-y} = -e^x + e^{-4} + 1$$

$$-y = \ln(-e^x + e^{-4} + 1)$$

$$y = -\ln(-e^x + e^{-4} + 1)$$

Feb 3-5:02 PM

4. $\frac{dy}{dx} = 3y$ *if* $y = 10$ *when* $x = 0$

Feb 3-5:03 PM

2005 Free Response #6

$$\cancel{dx}\ \frac{dy}{\cancel{dx}} = \frac{-xy^2}{2}\ \cancel{dx}; f(-1) = 2$$

x y

$$\int \frac{1}{y^2}\,dy = \int -\frac{1x}{2}\,dx$$

$$-\frac{1}{y} = -\frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$-\frac{1}{y} = -\frac{x^2}{4} + c$$

$$-\frac{1}{2} = -\frac{(-1)^2}{4} + c$$

$$-\frac{1}{2} = -\frac{1}{4} + c$$

$$-\frac{1}{4} = c$$

$$-4\left(-\frac{1}{y}\right) = \left(-\frac{x^2}{4} - \frac{1}{4}\right) - 4$$

$$\frac{\cancel{y} \cdot \frac{4}{\cancel{y}}}{x^2+1} = \frac{(\cancel{x^2+1})y}{\cancel{x^2+1}}$$

$$y = \frac{4}{x^2+1}$$

Feb 3-5:03 PM

Law of exponential change:

What does "exponential change" really mean anyway?

The law of exponential change (from a calculus perspective):

Feb 27-4:16 PM

growth constant

Solve $\frac{dy}{dt} = ky$ if $y = y_0$ when t=0

$\int \frac{1}{y}\, dy = \int k\, dt$

$\ln|y| = kt + c$

$\ln|y_0| = k(0) + c$

$\ln|y_0| = c$

$\ln|y| = kt + \ln|y_0|$

$y = e^{kt + \ln y_0}$

$y = e^{kt} \cdot \cancel{e^{\ln}}\, y_0$

$y = y_0 e^{kt}$

Algebraic Law of Exp Change

Feb 27-4:25 PM

The law of exponential change

Calc Version:

$\frac{dy}{dt} = ky$

see this go straight to

Algebra Version:

$y = y_0 e^{kt}$

Feb 27-4:28 PM

$k = -2.1$

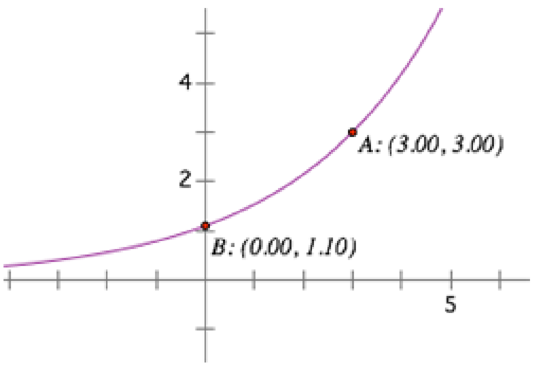
1. Solve $\frac{dy}{dt} = -2.1y \quad if\ y_0 = 100$

$y = y_0 e^{kt}$

$y = 100e^{-2.1t}$

Feb 27-4:29 PM

Find an equation in the form $y = y_0 e^{kt}$ for the following graph



Feb 27-4:31 PM

3. A certain bacteria grows at a rate proportional to the amount present. If after 3 hours there are 3000 bacteria and after 7 hours there are 5000 bacteria, what was the initial amount? How many will be present at 10 hours?

y_0 ?

$y = y_0 e^{kt}$

$3000 = y_0 e^{3(.1277)}$

$\frac{3000}{e^{3(.1277)}} = y_0$

$3000 = y_0 e^{3k}$ $5000 = y_0 e^{7k}$

$y_0 = \frac{5000}{e^{7k}}$

$3000 = \frac{5000}{e^{7k}} \left(e^{3k}\right)$

$3000 = \frac{5000 e^{3k}}{e^{7k}}$

$3000 = \frac{5000}{e^{4k}}$

$3000 e^{4k} = 5000$

$e^{4k} = 1.6$

$4k = \ln 1.667$

$k = \frac{\ln 1.667}{4} = .1277$

Feb 27-4:35 PM

4. A radioactive substance decays according to the equation $y = y_0 e^{-.06t}$. Find the half-life of the substance. How long before only 20% of the substance remains?

Feb 27-4:36 PM

5. The rate of change (in cubic inches per second) of the volume of water in a draining swimming pool is proportional to the amount present, according to the equation $\frac{dy}{dt} = -1.5y$.

The initial amount of water is 10,000 cubic inches. When will there be 100 cubic inches remaining?

Feb 27-4:37 PM

Newtons Law of Cooling $T - T_s = (T_0 - T_s)e^{-kt}$

6. A cinnamon eggo waffle is 200° when taken out of the toaster and set on your little sister's plate. After one minute, the eggo has cooled to 170°. If the eggo cools to 100° then the butter will no longer melt, and your little sister will throw a massive temper tantrum. You will then need make her new egos and eat the cold ones yourself. How much time do you have to butter the eggo? (Room temp is 70°)

Feb 27-4:42 PM

Jan 16-10:30 AM