6.2.1 U-Substitution

Consider $f'(x) = 2x \sin x^2$

What do you notice?

Can you find f(x)?

$f(x) = -\cos x^2$

Integration by substitution (U-Substitution)

$$\int 2x \sin x^2 \, dx$$

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Steps

$\int 2x \sin x^2 \, dx$

1. Identify the inside function $u = x^2$
2. Let u=inside function $du = 2x\,dx$
3. Find du $\int \sin u \, du$ $= -\cos u + c$
4. Get a "match" and substitute (If you can't get a "match" try another u) $= -\cos x^2 + c$
5. Integrate
6. Use substitution to get your answer back in terms of x
7. Check by differentiating

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Find each indefinite integral

$$\int e^{\cos x} \sin x\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= -\int e^u\, du$

$= -(e^u + c)$

$= -e^u + c$

$= -e^{\cos x} + c$

$$\int \sin^3 x \cos x\, dx$$

$= \int (\sin x)^3 \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx$

$= \int u^3\, du$

$= \frac{u^4}{4} + c$

$= \frac{\sin^4 x}{4} + c$

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$$\int x e^{x^2}\, dx$$

$u = x^2$

$\frac{du}{2} = \frac{2x\, dx}{2}$

$\frac{1}{2} du = x\, dx$

$= \frac{1}{2}\int e^u\, du$

$= \frac{1}{2}(e^u + c)$

$= \frac{1}{2} e^u + c$

$= \frac{1}{2} e^{x^2} + c$

$$\int x^3 \cos(3x^4)\, dx$$

$u = 3x^4$

$\frac{du}{12} = \frac{12x^3\, dx}{12}$

$\frac{1}{12}\int \cos u\, du$

$= \frac{1}{12}(\sin u + c)$

$= \frac{1}{12}\sin 3x^4 + c$

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$$\int \sqrt{\sec x}\,\sec x \tan x\,dx$$

$u = \sec x$

$du = \sec x \tan x\,dx$

$$\int \sqrt{u}\,du$$

$$\frac{2}{3}u^{3/2} + c$$

$$\frac{2}{3}(\sec x)^{3/2} + c$$

$$\int \frac{\ln^4 x}{2x}\,dx$$

$$= \frac{1}{2}\int \frac{\ln^4 x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{2}\int u^4\,du$$

$$= \frac{1}{2}\left(\frac{u^5}{5} + c\right)$$

$$= \frac{1}{10}\ln^5 x + c$$

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$$\int \frac{3\cancel{\csc^2\theta}}{\cot\theta}\,d\theta$$

$$3\int \frac{\csc^2\theta}{\cot\theta}\,d\theta = -3\int \frac{1}{u}\,du$$

$u = \cot\theta$

$du = -\csc^2\theta\,d\theta$

$-du = \csc^2\theta\,d\theta$

$$= -3\left(\ln|u| + c\right)$$

$$= -3\ln|\cot\theta| + c$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$-du = \sin x\,dx$

$$= -\int \frac{1}{u}\,du$$

$$= -\left(\ln|u| + c\right)$$

$$= -\ln|\cos x| + c$$

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$$\int \frac{\sqrt{\ln x}}{4x}\,dx$$

$= \frac{1}{4}\int \frac{\sqrt{\ln x}}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \frac{1}{4}\int \sqrt{u}\,du$

$= \frac{1}{4}\left(\frac{2}{3}u^{3/2} + c\right)$

$= \frac{1}{6}u^{3/2} + c$

$= \frac{1}{6}(\ln x)^{3/2} + c$

$$\int \frac{dx}{x^2+16}$$

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$$\int \frac{x}{x^2+1}\,dx$$

$u = x^2+1$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}(\ln|u| + c)$

$= \frac{1}{2}\ln(x^2+1) + c$

$$\int e^x \tan e^x\,dx$$

$u = e^x$

$du = e^x\,dx$

$= \int \tan u\,du$

$= \sec^2 u + c$

$= \sec^2 e^x + c$

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