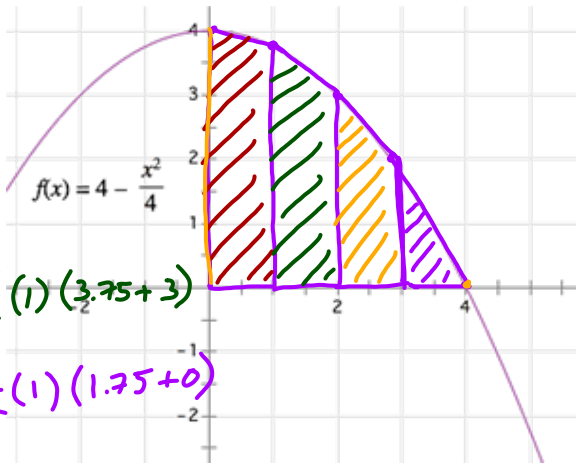
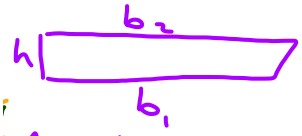
Calculus 5.5 Notes- The Trapezoidal Rule

Is there another way to approximate the area between the x-axis and $y = 4 - \frac{x^2}{4}$ over $[0, 4]$





$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(1)(4 + 3.75) + \frac{1}{2}(1)(3.75 + 3)$

$+ \frac{1}{2}(1)(3 + 1.75) + \frac{1}{2}(1)(1.75 + 0)$

$A = 10.5 \text{ units}^2$

A formula for the trapezoidal rule with uniform intervals:

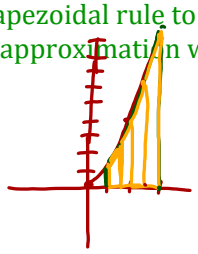
$$A_T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + \ldots + 2y_{n-1} + y_n)$$

Trapezoidal Rule! :)

Jan 31-9:53 PM

1. Use the trapezoidal rule to approximate the integral. Predict whether your approximation will be too high or too low. (4 subintervals)

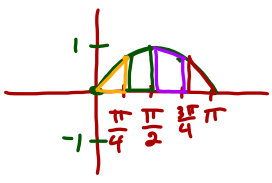
$\int_1^3 x^2\,dx$



$A_T = \frac{(\frac{1}{2})}{2}\left(1 + 2(\frac{9}{4}) + 2(4) + 2(\frac{25}{4}) + 9\right)$

$= 8.75 \text{ units}^2$

$\int_0^{\pi} \sin x\,dx$



$A_T = \frac{(\frac{\pi}{4})}{2}\left(0 + 2(\frac{\sqrt{2}}{2}) + 2(1) + 2(\frac{\sqrt{2}}{2}) + 0\right)$

$A_T = \frac{\pi}{8}(2 + 2\sqrt{2})$

$A_T = \frac{(1+\sqrt{2})\pi}{4} \text{ units}^2$

Jan 31-10:01 PM

A rectangular swimming pool is 40 feet wide and 45 feet long and has depth h(x). The following table shows the depths at 5 foot intervals. Estimate the volume of the pool using the trapezoidal rule.

Position (ft)	0	5	10	15	20	25	30	35	40	45
Depth (ft)	3	3.4	4	4.3	4.8	5.2	6.1	7.2	7.9	8

$A_T = \frac{5}{2}(3 + 2(3.4) + 2(4) + 2(4.3) + 2(4.8) + 2(5.2) + 2(6.1) + 2(7.2) + 2(7.9) + 8)$

$A = 242 \text{ ft}^2$

$242 \cdot 40 = 9680 \text{ ft}^3$

Jan 31-10:04 PM

The table below shows the velocity of a speed skater at 1 second intervals for 8 seconds. Use the trapezoidal rule to approximate the distance the skater traveled in the first 8 seconds.

area under rate of change curve

Time (sec)	0	1	2	3	4	5	6	7	8
Speed (miles/hr)	0	3	7	12	17	25	33	41	48

1/3600 hr

Jan 31-10:09 PM

AP type question (trapezoid formula doesn't work)

$60 + 70 + 30 = 160 \text{ units}^2$

x	2	5	7	8
$f(x)$	10	30	40	20

$A = \frac{1}{2}(3)(10+30) + \frac{1}{2}(2)(30+40) + \frac{1}{2}(1)(40+20)$

The function f is continuous on the closed interval [2,8] and has values that are given in the table above. Using the subintervals [2,5], [5,7], and [7,8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$