5.4 The Fundamental Theorem of Calculus

Part 1

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x)$$

$= f(x) - 0 = f(x)$

If $F(x) = \int_a^{g(x)} f(t)\,dt$, then

$F'(x) = f(g(x)) \cdot g'(x)$.

Jan 11-8:39 AM

Find $\frac{dy}{dx}$

1. $y = \int_\pi^x \cos t\,dt$

$y = \sin t \Big|_\pi^x = \sin x - \sin \pi$

$\frac{dy}{dx} = \cos x$

$y = \sin x - \sin \pi$

$\frac{dy}{dx} = \cos x$

2. $y = \int_{-1}^x (t^2 + t - 1)\,dt$

$y = \frac{t^3}{3} + \frac{t^2}{2} - t \Big|_{-1}^x$

$y = \left(\frac{x^3}{3} + \frac{x^2}{2} - x\right) - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 1\right)$

$y = \frac{x^3}{3} + \frac{x^2}{2} - x + \frac{1}{6}$

$\frac{dy}{dx} = x^2 + x - 1$

$\frac{dy}{dx} = x^2 + x - 1$

Jan 11-8:42 AM

3. $y = \int_4^{x^2} e^t \, dt$

$\frac{dy}{dx} = e^{x^2} \cdot 2x = 2xe^{x^2}$

4. $y = \int_{2x}^{x^2} \sin t \, dt$

$\frac{dy}{dx} = \sin x^2 \cdot 2x - \sin 2x \cdot 2$

$\frac{dy}{dx} = 2x \sin x^2 - 2 \sin 2x$

Jan 11-8:44 AM

5. $y = \int_{e^x}^{x^2} \ln t \, dt$

$\frac{dy}{dx} = \ln x^2 \cdot 2x - \ln e^x \cdot e^x$

$\frac{dy}{dx} = 2x \ln x^2 - xe^x$

or

$\frac{dy}{dx} = 4x \ln x - xe^x$

Jan 11-8:45 AM

Part 2

$$\int_a^b f(x)dx = [F(x)]^b_a = F(b) - F(a)$$

6. $\int_1^4 x^{\frac{2}{3}} dx$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}}$

$= \frac{3}{5} x^{\frac{5}{3}} \Big|_1^4$

$= \left(\frac{3}{5}(4)^{\frac{5}{3}}\right) - \left(\frac{3}{5}(1)^{\frac{5}{3}}\right)$

$= \frac{3}{5}\left(4^{\frac{5}{3}} - 1\right)$

7. $\int_0^2 \frac{\sqrt{x}+3}{\sqrt{x}} dx$

$1 + \frac{3}{\sqrt{x}}$

$= \int_0^2 1 + 3x^{-\frac{1}{2}} dx$

$= x + 6x^{\frac{1}{2}} \Big|_0^2$

$= 2 + 6(2)^{\frac{1}{2}} - 0$

$= 2 + 6\sqrt{2}$

Jan 11-8:46 AM

Find the total area between the curve and the x-axis analytically. Support your answers graphically.

8. $y = 4 - x^2$, $[0,3]$

$= \int_0^3 4 - x^2 \, dx$

$= \int_0^2 4 - x^2 \, dx + \left| \int_2^3 4 - x^2 \, dx \right|$

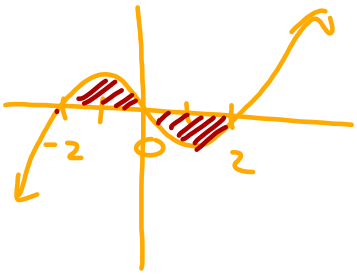
$= 4x - \frac{x^3}{3} \Big|_0^2 + \left| \left[4x - \frac{x^3}{3}\right]_2^3 \right|$

$= \left(8 - \frac{8}{3}\right) + \left(3 - \left(8 - \frac{8}{3}\right)\right)$

$= \frac{23}{3}$

Jan 11-8:48 AM

9. $y = x^3 - 4x, \ [-2,2]$



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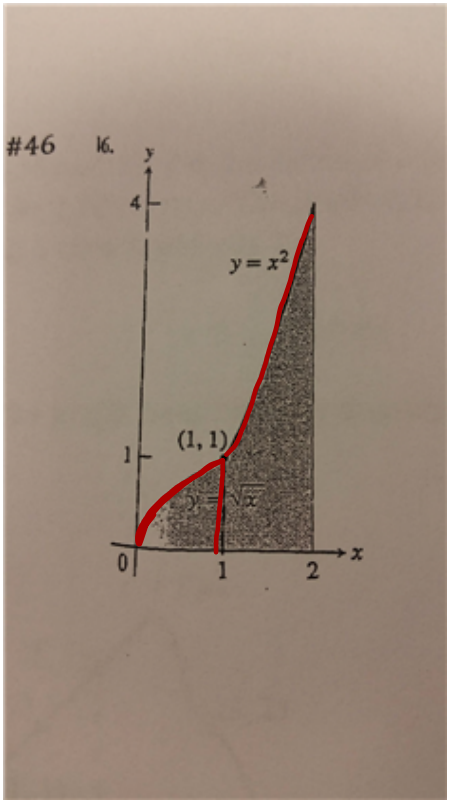

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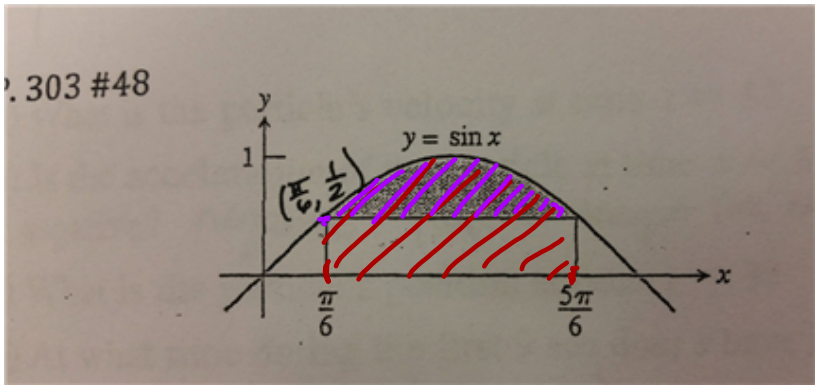
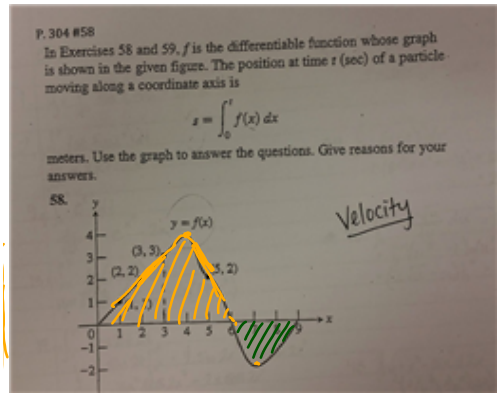


Jan 11-9:00 AM



a) What is the particle's velocity at time t=5?

2

b) Is the acceleration of the particle at time t=5 positive or negative?

neg.

c) What is the particle's position at time t=3?

4.5

d) At what time during the first 9 sec does s have its largest value?

t=6 sec.

e) Approximately when is the acceleration zero?

t=4,7 sec.

f) When is the particle moving toward the origin? away from the origin?

away 0-6

toward 6-9

g) On which side of the origin does the particle lie at time t=9?

pos.

Jan 11-9:00 AM