5.3.1 Rules for Integrals

2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

1: $\int_a^a f(x)dx = 0$

~~RS#74~~ 6: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

~~RS#73~~ 3: $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

5. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Given

$$\int_3^7 f(x)dx = 6 \qquad \int_7^{10} f(x)dx = -1$$

$$\int_7^{10} g(x)dx = 4 \qquad \int_3^5 f(x)dx = 2$$

Find each of the following integrals

1. $\int_7^3 f(x)dx$

$= -\int_3^7 f(x)\,dx$

$= -6$

2. $\int_2^2 f(x)dx$

$= 0$

Given

$$\int_3^7 f(x)dx = 6 \qquad \int_7^{10} f(x)dx = -1$$

$$\int_7^{10} g(x)dx = 4 \qquad \int_3^5 f(x)dx = 2$$

3. $\int_7^{10} (2f(x) + 3g(x))dx$

$= 2\int_7^{10} f(x)\,dx + 3\int_7^{10} g(x)\,dx$

$= 2(-1) + 3(4)$

$= 10$

4. $\int_{10}^{3} f(x)dx$

$= -\int_3^{10} f(x)\,dx$

$= -\left[\int_3^7 f(x)\,dx + \int_7^{10} f(x)\,dx\right]$

$= -[6 + (-1)]$

$= -5$

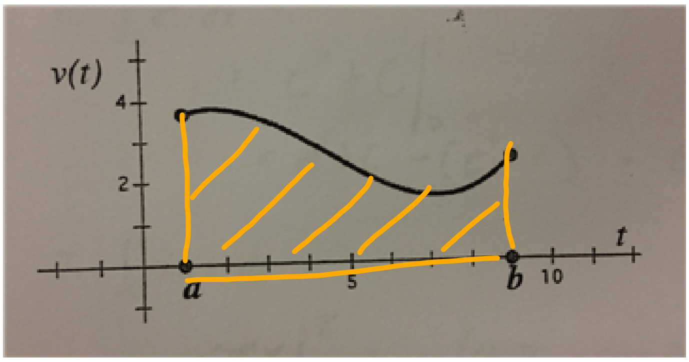
5. $\int_5^7 f(x)dx$

$= \int_3^7 f(x)dx - \int_3^5 f(x)\,dx$

$= 6 - 2$

$= 4$

Consider the following velocity graph of the motion of a particle.



Using an integral, express the distance traveled by the particle: $\int_a^b v(t)\,dt$

Find another way to express the distance traveled by the particle (using s(t)).

$s(b) - s(a)$

Put the two results above together

$\int_a^b v(t)\,dt = s(b) - s(a)$

What is the significance of the above equation?

Evaluate the following integrals analytically.

→ algebra

6. $\int_0^1 e^x \, dx$

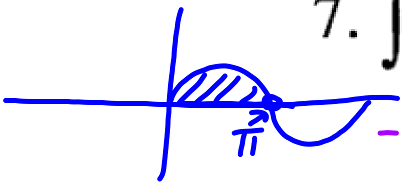
$e^x + c \Big|_0^1$

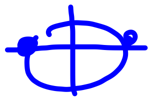
$(e^1 + c) - (e^0 + c)$

$e + \cancel{c} - 1 - \cancel{c}$

$e - 1$

7. $\int_0^\pi \sin x \, dx$





$-\cos x \Big|_0^\pi$

$(-\cos \pi) - (-\cos 0)$

$1 - -1 = 2$

8. $\int_1^e \frac{1}{x} \, dx$

$\ln x \Big|_1^e$

$(\ln e) - (\ln 1)$

$= 1 - 0$

$= 1$

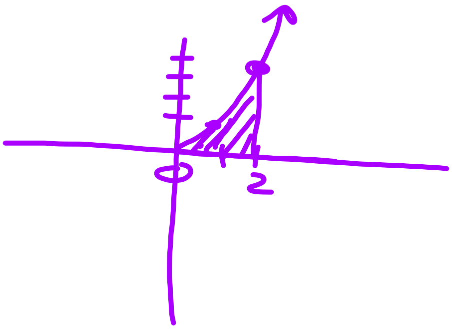
9. $\int_0^4 x^3 \, dx$

$= \frac{1}{4} x^4 \Big|_0^4$

$= \left(\frac{1}{4}(4)^4\right) - \left(\frac{1}{4}(0)^4\right)$

$= 64 - 0 = 64$

Find the area between $y = x^2$ and the x-axis over [0,2]



$$\int_0^2 x^2 \, dx$$

$$= \frac{1}{3}x^3 \Big|_0^2$$

$$= \left(\frac{1}{3}(2)^3\right) - \left(\frac{1}{3}(0)^3\right) = \frac{8}{3}$$