5-3 Solving Exponential and Logarithmic equations

Objectives:

5-3a: I can solve exponential and logarithmic equations graphically.

5-3b: I can solve exponential and logarithmic equations algebraically.

Jan 3-2:07 PM

Inverses

Addition/Subtraction	Natural Log/e^	Common Log/10^	Log base b/b^
$x-5=10$ $+5 \quad +5$ $x=15$	$\ln e^x = \ln 5$ $x = \ln 5$	$\log 10^x = \log 100$ $x = 2$	$2^x = 16$ $\log_2 \quad \log_2$ $x = 4$
$x+7=21$ $-7 \quad -7$ $x=14$	$\ln x = 7$ $e \quad e$ $x = e^7$	$\log x = 3$ $10 \quad 10$ $x = 1000$	$\log_3 x = 4$ $3 \quad 3$ $x = 81$

Jan 28-12:33 PM

Solve the following WITHOUT a calculator

$\log_4(x-1)=1$

$4^{\log_4(x-1)}=4^1$

$x-1=4$

$x=5$

$\log(4x)=2$

$10^{\log(4x)}=10^2$

$4x=100$

$x=25$

You try:

$\log_3(2x-4)=3$

$3^{\log_3(2x-4)}=3^3$

$2x-4=27$

$2x=31$

$x=15.5$

Jan 3-3:11 PM

Solve the following WITHOUT a calculator

$\log(x+2)+3=5$

$-3 \quad -3$

$10^{\log(x+2)}=10^2$

$x+2=100$

$x=98$

$\ln(x+12)=3\ln 2$

$=\ln 2^3$

$e^{\ln(x+12)}=e^{\ln 8}$

$x+12=8$

$x=-4$

$\ln=\log_e$

$e\approx 2.7$

Solve the following WITHOUT a calculator

$4^{3x-1} - 2 = 14$

$+2 \quad +2$

$\log_4 4^{3x-1} = \log_4 16$

$3x - 1 = 2$

$3x = 3$

$x = 1$

$2(5)^{4x} = 250$

$\div 2 \quad \div 2$

$\log_5 5^{4x} = \log_5 125$

$4x = 3$

$x = \frac{3}{4}$ or .75

$e^{2x+1} + 3 = 4$

$-3 \quad -3$

$\ln e^{2x+1} = \ln 1$

$2x + 1 = 0$

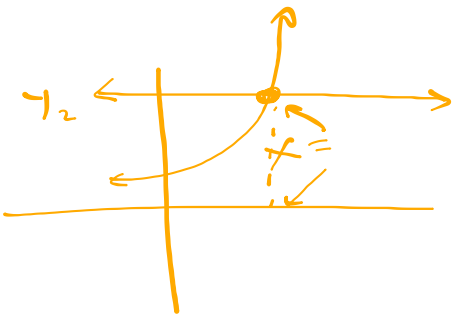
$2x = -1$

$x = -\frac{1}{2}$ or $-.5$

RECALL: Solving Graphically

$275e^{0.06x} = 1000$

$y_1 = 275e^{0.06x} \quad y_2 = 1000$



$10^{2x} = 1500$

$y_1 =$ $y_2 =$

Now you try...solve the exponential & logarithmic equations graphically.

$20^{2r} = 56$

$y_1 =$ $y_2 =$

$e^{.23x} = 1.99$

$y_1 =$ $y_2 =$

Solve the following equations with a calculator.

$\frac{10}{5} = \frac{5e^{4x}}{5}$

calc

$\ln 2 = \ln e^{4x}$

$\frac{.69}{4} = \frac{4x}{4}$

$x = .17$

$5^{x-1} - 4 = 7$

$6^{3x} = 12$

$4\ln(x+7) - 5 = 1$

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

P is the principal

r is the annual interest rate

n is the number of compounding periods per year

t is the time in years

Feb 15-11:03 AM

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

Continuous Compounding Formula

If *P* dollars are invested at an interest rate *r*, that is compounded continuously, then the amount, *A*, of the investment at time *t* is given by

$$A(t) = Pe^{rt}$$

P is principal (initial value)

r is interest rate

t is time (in years)

Feb 15-11:03 AM

How long (t) will it take for a $250 (P) initial investment in an account that pays 4.5% (r) compounded continuously to grow to $750?

A(t)

$\frac{750}{250} = \frac{250e^{.045t}}{250}$

$\ln 3 = \ln e^{.045t}$

$\frac{1.09}{.045} = \frac{.045t}{.045}$

$t \approx 24.2$ yrs

Suppose that $250 is deposited into an account that pays 4.5% compounded quarterly. Solve for t to find how long it will take for the account to contain at least $500.

Jan 3-3:03 PM

$\log_2 7$ Log with base other than 10 or e in calc.

1) Math button
2) ↑ until "log base"
3) Enter

$\log_{\square} \square$

$2x^2 + 4x + 7 = 6$

$2(x^2 + 2x + \frac{7}{2}) = 6$

Complete: $\frac{2}{2} = 1$, $(1)^2 = 1$

$2(x^2 + 2x + 1 + \frac{7}{2} - 1) = 6$

$\frac{2((x+1)^2 + \frac{5}{2})}{2} = \frac{6}{2}$

$(x+1)^2 + \frac{5}{2} = 3$

$\sqrt{(x+1)^2} = \sqrt{\frac{1}{2}}$

$\frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} =$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$x + 1 = \pm \frac{\sqrt{2}}{2}$

$x = -1 \pm \frac{\sqrt{2}}{2}$

Comparing acidity: $pH = -\log[H^+]$

$[H^+]$ = hydrogen-ion concentration

Vinegar has a pH of 2.4. What is it's hydrogen ion concentration?

$\frac{2.4}{-1} = \frac{-\log(H^+)}{-1}$

$10^{-2.4} = 10^{\log(H^+)}$

$H^+ = 10^{-2.4}$

Baking soda has a pH of 8.4. What is it's hydrogen ion concentration?

$8.4 = -\log(H^+)$

$-8.4 = \log(H^+)$

$H^+ = 10^{-8.4}$

Which has a higher hydrogen ion concentration?

Vinegar