5-1 Defining and Evaluating Logarithms

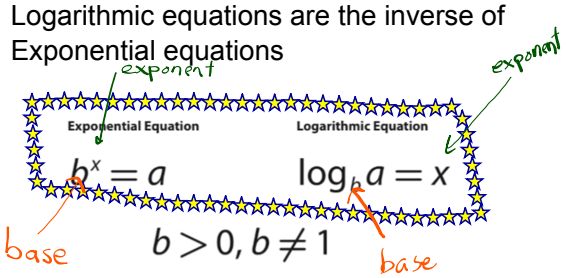
Objectives:

5-1a: I can convert between logarithm and exponential form.

5-1b: I can evaluate a logarithmic expression.

Nov 11-7:56 AM

Logarithmic equations are the inverse of Exponential equations



Apr 26-12:54 PM

Examples

Exponential Equation	Logarithmic Equation
$4^3 = 64$	$\log_4 64 = 3$
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$
$3^5 = 243$	$\log_3 243 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$
$\left(\frac{3}{4}\right)^t = s$	$\log_{\frac{3}{4}} s = t$
	$\log_{\frac{1}{5}} v = w$

Nov 11-7:57 AM

$e = 2.72$ $\ln = \log_e$

The natural logarithm:

$y = \ln x$ is equivalent to $x = e^y$

$\ln x = y$ $e^y = x$

The common logarithm: (base 10)

$y = \log x$ is equivalent to $x = 10^y$

$\log x = y$ $10^y = x$

Apr 27-9:51 AM

Exponential Equation	Logarithmic Equation
$e^5 \approx 148.4$	$\ln 148.4 \approx 5$
$e^{1.8} \approx 6$	$\ln 6 \approx 1.8$
$10^5 = 100{,}000$	$\log 100{,}000 = 5$
$10^3 = 1000$	$\log 1{,}000 = 3$

Nov 11-8:04 AM

If $f(x) = \log_{10} x$, find $f(1000)$, $f(0.01)$, and $f(\sqrt{10})$.

$\log_{10} 1000 = ?$

$10^{?} = 1000$ $\quad ? = 3$

If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$, and $f\left(2\sqrt{2}\right)$.

$\log_{1/2} 4 = ?$

$\left(\frac{1}{2}\right)^{?} = 4$ $\quad ? = -2$

Nov 11-8:36 AM

Find the exact value without a calculator

$4^1 = 4$ $\quad 4^{-1} = \frac{1}{4}$

$4^2 = 16$ $\quad 4^{-2} = \frac{1}{16}$

$\log_2 32 = 5$

$2^{?} = 32$

$\log_4 \frac{1}{16} = -2$

$\log 10{,}000{,}000$

7

$\log .00001$

-5

Apr 27-9:40 AM

You try

$\log_5 25 = 2$

$5^{x} = 25$

$\log_2 \frac{1}{8} = -3$

$\log 1000$

$= 3$

$\log .001$

$= -3$

Apr 27-9:41 AM

The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[\text{H}^+]}$ where $[\text{H}^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from 1.58×10^{-8} moles per liter to 6.31×10^{-8} moles per liter. What is the range of the pH for a typical swimming pool?

$PH = \log \frac{1}{1.58 \times 10^{-8}} = 7.8$

$PH = \log \frac{1}{6.31 \times 10^{-8}} = 7.2$

7.2 to 7.8

Nov 14-10:42 AM

The intensity level L (in decibels, dB) of a sound is given by the formula $L = 10 \log \frac{I}{I_0}$ where I is the intensity (in watts per square meter, W/m^2) of the sound and I_0 is the intensity of the softest audible sound, about 10^{-12} W/m^2. What is the intensity level of a rock concert if the sound has an intensity of 3.2 W/m^2?

$L = 10 \log \frac{3.2}{10^{-12}} \approx 125$ dB

Nov 14-10:47 AM