4.6 Related Rates

Objectives:

- I can write a model to represent a situation

- I can find the rate of change of one value given another

Oct 18-1:48 PM

Process:

1- Understand the problem

2- Write down what you know/need to know. Identify variables and constants.

3- Write an equation relating the variables.

4- Differentiate *with respect to t.* $\frac{d}{dt}$

5- Solve for the unknown value.

Dec 2-11:18 AM

1. The radius of a sphere is changing at a rate of 2 in/sec.
A) How fast is the volume of the sphere changing when the radius is 3 inches?

$\frac{dr}{dt} = 2 \text{ in/s}$

$r = 3$ in.

$\frac{dV}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (3)^2 (2) = 72\pi \text{ in}^3/\text{s}$

B) How fast is the surface area of the sphere changing when the radius is 3 in.?

$\frac{d(SA)}{dt} = ?$

$SA = 4\pi r^2$

$\frac{d(SA)}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(2)$
$= 48\pi \text{ in}^2/\text{s}$

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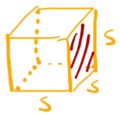
2. The sides of a cube are increasing at a rate of 4 in/sec.
A) How fast is the volume increasing when the length of a side is 2 inches?

$s = 2$ in.



$\frac{ds}{dt} = 4 \text{ in/s}$

$\frac{dV}{dt} = ?$

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3(2)^2(4) = 48 \text{ in}^3/\text{s}$

B) How fast is the surface area increasing when the length of a side is 3 inches?

$\frac{d(SA)}{dt} = ?$

$s = 3$ in.

$SA = 6s^2$

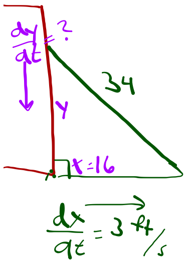
$\frac{d(SA)}{dt} = 12s \frac{ds}{dt} = 12(3)(4)$
$= 144 \text{ in}^2/\text{s}$

Dec 2-11:20 AM

3. A 34 foot ladder is sliding down the side of a building. The base is moving away from the building at a rate of 3 ft/sec.

A) How fast is the top of the ladder falling down when the base is 16 feet away?

variable



$16^2 + y^2 = 34^2$

$y = 30$

$x^2 + y^2 = 34^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

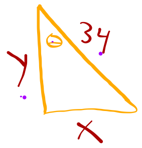
$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

$y\frac{dy}{dt} = -x\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y}$

$\frac{dy}{dt} = \frac{-(16)(3)}{30} = -1.6$ ft/s

B) How fast it the angle with the building changing when the base is 16 feet away? $\frac{d\theta}{dt}$



$\sin\theta = \frac{x}{34}$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{34}\frac{dx}{dt}$

$\cos\theta = \frac{30}{34}$

$\left(\cancel{\frac{34}{30}}\right)\left(\cancel{\frac{30}{34}}\right)\frac{d\theta}{dt} = \frac{1}{\cancel{34}}(3)\left(\frac{\cancel{34}}{30}\right)$

$\frac{d\theta}{dt} = .1$ rad/s

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4. Melanie is flying a kite at a height of 400 feet. The wind is carrying the kite away at a horizontal speed of 20 ft/s. How fast must Melanie release the string when the kite is 500 feet away in order to maintain the height of 400 ft?

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5. Two cars are moving toward the same intersection on perpendicular streets. Car A is travelling at 30 mi/h and car B at 45 mi/h. How fast is the distance between them changing when A is ½ mile away and B is ¼ mile away.

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6. A spotlight is moving in a linear direction at a speed of $\frac{\pi}{4}$. The light beam is tracing across uniform cloud cover that is 500ft high. What is the speed of the beam on the clouds when the spotlight forms an angle of 60° with the ground?

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7. A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow as shown in the figure. The rope is hauled in at a rate of 2 ft/sec.

A) How fast is the boat approaching when 10 ft of rope are out?

B) At what rate is the angle θ changing at that moment?

$\frac{dz}{dt} = 2 \text{ ft/s}$

$\frac{dx}{dt} = ?$

$z = 10$ ft.

$x^2 + 36 = z^2$

$\cancel{2}x \frac{dx}{dt} = \cancel{2}z \frac{dz}{dt}$

$\frac{dx}{dt} = \frac{z \frac{dz}{dt}}{x}$

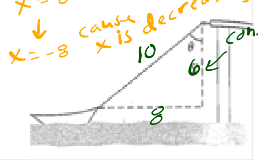
$\frac{dx}{dt} = \frac{(10)(2)}{(-8)} = -2.5 \text{ ft/s}$

$x^2 + 36 = 100$

$x = 8$

$\downarrow$

$x = -8$ cause x is decreasing



$\tan\theta = \frac{x}{6}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{6} \frac{dx}{dt}$

$\left(\frac{100}{36}\right) \frac{d\theta}{dt} = \frac{1}{6}(-2.5)$

$\frac{d\theta}{dt} = \frac{-2.5}{\cancel{6}} \cdot \frac{\cancel{36}\ \cancel{6}\ 3}{\cancel{100}\ 50}$

$\boxed{\frac{d\theta}{dt} = -.15 \text{ rad/s}}$

Dec 2-11:27 AM

8. You are videotaping a race from a stand 132 feet from the track, following a car that is moving at 180 miles per hour (264 ft/sec). About how fast will your camera angle be changing when the car is right in front of you? A half second later?

Dec 2-11:28 AM