4.5/4.2 Linearization, Differentials, and Antiderivatives

Objectives:

- I can find a general antiderivative
- I can evaluate using differentials
- I can find the linearization of a function

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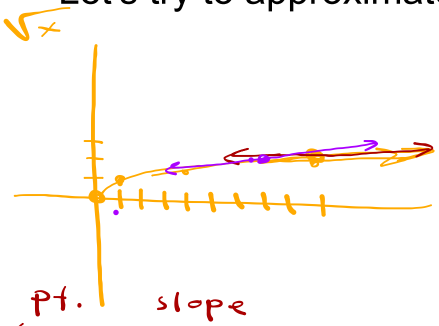
Let's try to approximate $\sqrt{7}$ without a calculator



$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

pt. $(9, 3)$ slope $\frac{1}{6}$

$y = \frac{1}{6}(x-9)+3$

$y = \frac{1}{6}(7-9)+3$

$y = -\frac{1}{3}+3 = \frac{8}{3} \approx 2.666666666666666666666666$

$\sqrt{7} \approx 2.646$

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The linearization of f at a is given by:

$$L(x) = \underbrace{f'(a)}_{\text{slope}}(x - \underbrace{a}_{x_1}) + \underbrace{f(a)}_{y_1}$$

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Use an appropriate linearization to approximate $\sqrt{110}$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(100) = \frac{1}{20}$

pt. $(100, 10)$ slope $\frac{1}{20}$

$y = \frac{1}{20}(x - 100) + 10$

$y = \frac{1}{20}(110 - 100) + 10$

$y = \frac{1}{2} + 10 = 10.5$

calc $\sqrt{110} \approx 10.488$

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Find the linearization L(x) of f(x) at x=a

$$f(x) = x + \frac{1}{x}, \;\; a = 1$$

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Differentials

$\frac{dy}{dx}$ is a notation that represents the derivative.

Separately, dy and dx are called differentials.

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Find dy. Then evaluate dy for the given values.

1. $y = \frac{x}{x^2 - 1}$, $x = 3, dx = .01$

$x^2 - 1 \quad -2x^2$

$dy = \frac{(x^2-1)(1) - x(2x)}{(x^2-1)^2} dx$

$dy = \frac{-x^2-1}{(x^2-1)^2} dx$

dy

$dy = \frac{-(3)^2 - 1}{((3)^2-1)^2}(.01) = \frac{-10}{64}(.01) = -.0015$

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2. $y = x\sqrt{x-1}$, $x = 2, dx = .005$

$dy = x\left(\frac{1}{2\sqrt{x-1}}\right) + \sqrt{x-1}(1)(dx)$

$dy = \left(\frac{x}{2\sqrt{x-1}} + \sqrt{x-1}\right)dx$

$dy = \left(\frac{\cancel{2}}{\cancel{2}\sqrt{2-1}} + \sqrt{2-1}\right)(.005)$

$dy = .01$

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If $f'(x) = \underline{2x}^{1} + 4x^{0}$ can we find f(x)??

$$f(x) = \frac{2x^{1+1}}{1+1} + \frac{4x^{0+1}}{0+1}$$

$$f(x) = x^2 + 4x + c$$

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Find each general antiderivative

3. $f'(x) = x^2 - 2x$

$$f(x) = \frac{x^3}{3} - x^2 + C$$

4. $f'(x) = \sec^2 x$

$$f(x) = \tan x + c$$

5. $f'(x) = \dfrac{1}{x+6},\ x > -6$

$$f(x) = \ln(x+6) + c$$

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Consider $f'(x) = 2x + 4$ again. When finding f(x) what information would we need to find C?

$(x, f(x))$

$f(x) = x^2 + 4x + C$

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Find the particular antiderivative that passes through the point P.

6. $f'(x) = 3x \quad P(\overset{x}{1}, \overset{f(x)}{5})$

$f(x) = \frac{3x^2}{2} + C$

$5 = \frac{3(1)^2}{2} + C$

$5 = \frac{3}{2} + C$

$C = 5 - \frac{3}{2} = 3\frac{1}{2}$ or 3.5 or $\frac{7}{2}$

$f(x) = \frac{3x^2}{2} + \frac{7}{2}$

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7. $f'(x) = \dfrac{1}{3x^{\frac{2}{3}}}$ $\quad P(8,3)$

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8. $f'(x) = x^2 + 2 + \sin x \quad P(0,5)$

$f(x) = \frac{x^3}{3} + 2x - \cos x + C$

$5 = \frac{0^3}{3} + 2(0) - \cos(0) + C$

$5 = -1 + C$

$C = 6$

$f(x) = \frac{x^3}{3} + 2x - \cos x + 6$

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