4.4 Optimization

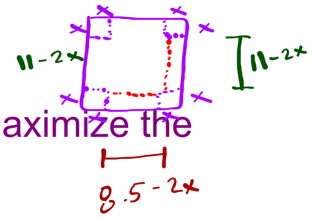
Objectives:

- I can write a model to represent a situation

- I can solve a model to maximize or minimize a value

Nov 5-9:54 AM

Consider an open box made by cutting congruent squares out of the corners of an 8 1/2 x 11 in sheet of paper.



What size square could be cut out to maximize the volume?

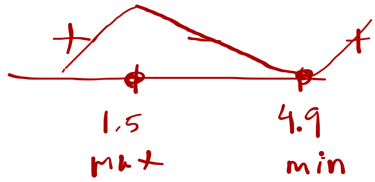
$V = lwh$

$V = (11-2x)(8.5-2x)x$

$V = 93.5x - 39x^2 + 4x^3$

$V' = 93.5 - 78x + 12x^2 = 0$

solve(93.5 − ... + 12x² = 0, x)



$x = 1.5854, 4.9146$

square is 1.5854 in. x 1.5854 in.

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Process for optimizing

1- Understand the problem- Draw a diagram and assign variables.

2- Write a model (function) to represent the problem

-Start with the equation of what you are trying to maximize

- Use substitution to get the equation as a function of only one variable.

3- Identify critical points and closed endpoints as candidates.

4- Test critical points to determine max/min

- Plug into function or

- First derivative test or

- Second derivative test

5- Interpret/state solution

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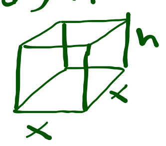
1. You are designing an open box with a square base and a required volume of 108 cubic inches. What dimensions would minimize the materials needed?

$V = lwh \qquad h = \frac{108}{x^2}$

$V = (6)(6)h = 108 \qquad h = 3$



6in × 6in × 3in

$SA = x^2 + 4xh$

$SA = x^2 + 4\cancel{x}\left(\frac{108}{x^{\cancel{2}}}\right)$

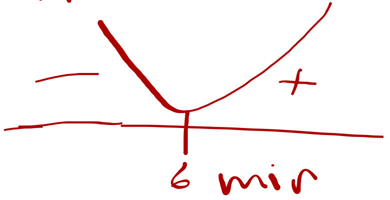
$SA = x^2 + \frac{432}{x}$

$\frac{d}{dx}\left(x^2 + \frac{432}{x}\right)$

$SA' = 2x - \frac{432}{x^2} = 0$

solve(ans = 0, x)

$x = 6$



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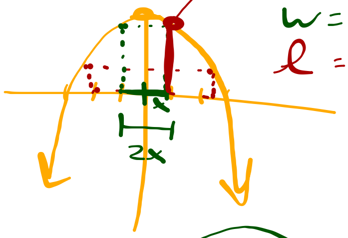
2. A rectangle is inscribed between $y=-x^2+9$ and the x-axis. Find the maximum area of the rectangle.

$(x, -x^2+9)$



$w = 2x$

$l = -x^2+9$

$A = lw$

$A = (-x^2+9)(2x)$

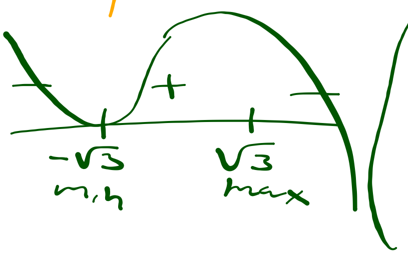
$A = -2x^3 + 18x$

$A' = -6x^2 + 18 = 0$

$6x^2 = 18$

$x^2 = 3$

$x = \pm\sqrt{3}$



Max at $x = \sqrt{3}$

$-2(\sqrt{3})^3 + 18\sqrt{3}$

$= 20.7846 \text{ units}^2$

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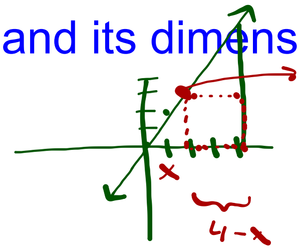
You try with a partner!

A rectangle is inscribed between $y=2x$, when $0 \le x \le 4$ and the x-axis. Find the maximum area of the rectangle, and its dimensions.

$(x, 2x)$



$A = lw$

$A = (4-x)(2x)$

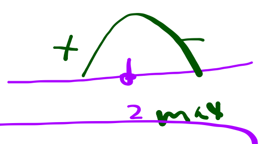
$A = 8x - 2x^2$

$A' = 8 - 4x$

$8 - 4x = 0$

$4x = 8$

$x = 2$



$A(2) = 8(2) - 2(2)^2$

$= 16 - 8$

$= 8$

$l = 4 - x = 4 - 2 = 2 \text{ units}$

$w = 2x = 2(2) = 4 \text{ units}$

$A = 8 \text{ units}^2$

3. A rectangle is inscribed between y = sinx and the x-axis over [0,π]. Find the maximum area of the rectangle.

Nov 5-10:01 AM

4. Design a 2-liter can with minimum surface area.

$2 = \pi r^2 h$ $\qquad h = \frac{2}{\pi r^2} = 13.6558$ cm

h $SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\cancel{\pi} r\left(\frac{2}{\cancel{\pi} r^2}\right)$

$SA = 2\pi r^2 + \frac{4}{r}$

$SA' = 4\pi r - \frac{4}{r^2} = 0$

$r^2(4\pi r) = \frac{4}{\cancel{r^2}}\ \cancel{r^2}$

$4\pi r^3 = 4$ $\qquad r = \sqrt[3]{\frac{1}{\pi}}$

$\pi r^3 = 1$

$r^3 = \frac{1}{\pi}$ $\qquad r = .6828$ dm

$r = 6.8$ cm

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5. What are the dimensions of the lightest open top right cylindrical can that will hold 2197 cubic cm?

Dec 2-10:44 AM

6. Suppose $r(x) = \dfrac{x^2}{x^2+1}$ represents revenue and $c(x) = \dfrac{(x-1)^2}{3} - \dfrac{1}{3}$ represents cost, with x measured in thousands of units. What is the production level that maximizes profit?

Dec 2-10:48 AM

7. Bobby is 3 miles off shore in a boat and wants to reach a campsite that is 5 miles down a straight shoreline from the point nearest to his boat. He can row 3 mph and jog 4 mph. How far from camp should he land his boat to minimize the time to reach the camp?

Dec 2-10:52 AM

8. How close does the semicircle $y=\sqrt{16-x^2}$ come to the point $(1,\sqrt{3})$

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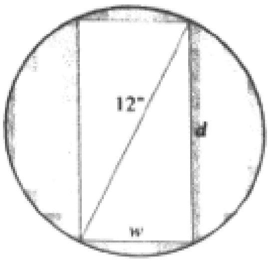

38. ***Stiffness of a Beam*** The stiffness S of a rectangular beam is proportional to its width times the cube of its depth.

(a) Find the dimensions of the stiffest beam that can be cut from a 12-in. diameter cylindrical log.

(b) Writing to Learn Graph S as a function of the beam's width w, assuming the proportionality constant to be $k = 1$. Reconcile what you see with your answer in part (a).

Dec 2-10:55 AM