4.3 Derivative Tests

Objectives:

- I can find extremes of a function using the first derivative test

- I can determine concavity of a function using the second derivative

- I can use the second derivative test to find extreme values

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The First Derivative Test (for local extrema)

*Find all extreme values and increasing/decreasing intervals

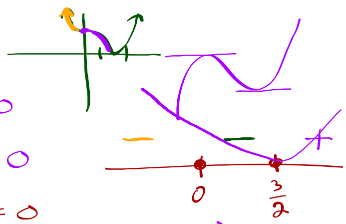
$f(x) = x^4 - 2x^3 + 2$

$f'(x) = 4x^3 - 6x^2 = 0$

$2x^2(2x-3) = 0$

$2x^2 = 0$ $\quad$ $2x - 3 = 0$

$x = 0$ $\quad$ $x = \frac{3}{2}$



Inc: $(\frac{3}{2}, \infty)$

Dec: $(-\infty, 0) \cup (0, \frac{3}{2})$

What does f(x) represent?

function val or y val

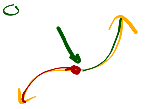
What does f '(x) represent?

slope → inc and dec

What does f ''(x) represent?

concavity (up or down)

$f'' = 0$



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Concavity and the Second Derivative

Concave up:

$f'' > 0$

pos

Concave down:

$f'' < 0$

neg

Inflection point:

where changes from conc. up to conc. down

a.k.a. f'' changes sign

Concavity Test

sign chart w/ 2^{nd} deriv

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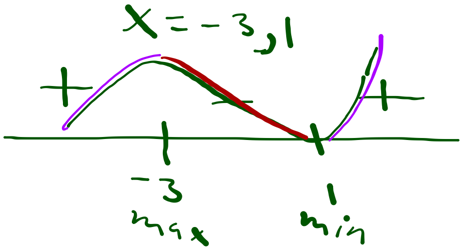
Analyze the function using the first derivative test and the concavity test

1. $y = \frac{1}{3}x^3 + x^2 - 3x + 2$

$y' = x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$



$y'' = 2x + 2 = 0$

$2x + 2 = 0$

$x = -1$

− | + at -1

inc: $(-\infty, -3) \cup (1, \infty)$

dec: $(-3, 1)$

max: $x = -3$

min: $x = 1$

conc up: $(-1, \infty)$

conc down: $(-\infty, -1)$

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Analyze the function using the first derivative test and the concavity test

$$y=-2x^3+6x^2-3$$

$y' = -6x^2 + 12x = 0$

$-6x(x-2) = 0$

$x = 0, 2$

$-$ $+$ $-$

0 2

$y'' = -12x + 12$

$-12x + 12 = 0$

$x = 1$

$+$ $-$

1

inc: $(0, 2)$

dec: $(-\infty, 0) \cup (2, \infty)$

max: $x = 2$

min: $x = 0$

conc up: $(-\infty, 1)$

conc down: $(1, \infty)$

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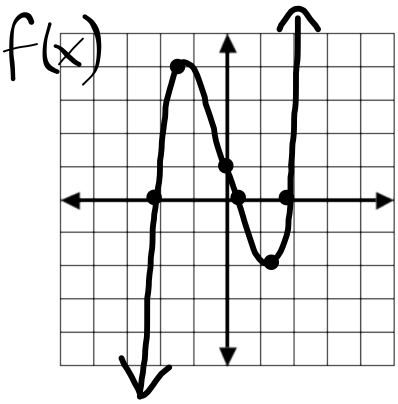
2. Use the given graph of f (x) to estimate the following.



a) Increasing

b) Decreasing

c) Local Extrema

d) Inflection Points

e) Concave up

f) Concave down

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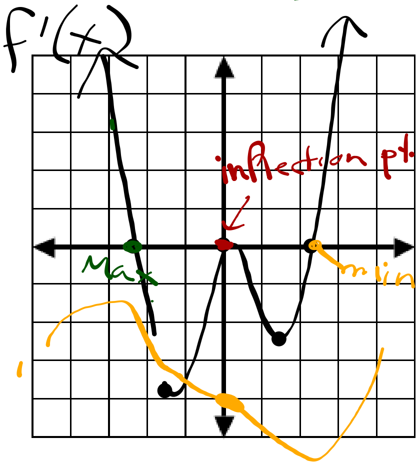
Use the given graph of f '(x) to estimate the following on f(x).



a) Increasing $f'(x) > 0$

$(-\infty, -2.3) \cup (2.3, \infty)$

b) Decreasing $f'(x) < 0$

$(-2.3, 0) \cup (0, 2.3)$

c) Local Extrema

$x = -2.3$ is a max

$x = 2.3$ is a min

d) Inflection Points

$x = 0$

e) Concave up $f''(x) > 0$

$(0, \infty)$

f) Concave down $f''(x) < 0$

$(-\infty, 0)$

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How do we find a critical point?

$y' = 0$ or DNE

How do we know if a critical point is an extreme value?

- f' sign changes
- 2^{nd} deriv test

How do we find an inflection point?

sign change f''

($y'' = 0$ sign change)

What is an inflection point?

Concavity changes

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Second Derivative Test For Extrema

If f '(c) =0 and f "(c) <0, then f has a local max at x=c

If f '(c)=0 and f "(c) >0, then f has a local min at x=c

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Use the second derivative test to final all max/min values of each function

1. $g(x)=-x^3+9x$

$g'(x) = -3x^2 + 9 = 0$

$-3x^2 = -9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$g''(x) = -6x$

$g''(\sqrt{3}) = -6\sqrt{3} = \text{neg}$

so $\sqrt{3}$ is a max

$g''(-\sqrt{3}) = -6(-\sqrt{3})$

$= \text{pos}$

so $-\sqrt{3}$ is a min

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2. $f(x) = x^5 - 80x + 100$

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3. $y = xe^x$

$y' = xe^x + e^x$

$y'' = xe^x + e^x + e^x$

$xe^x + e^x = 0$

$e^x(x+1) = 0$

$e^x = 0$ und $\quad x+1=0$, $x=-1$

$y'' = 2e^x + xe^x$

$y''(-1) = 2e^{-1} + (-1)e^{-1}$

$= e^{-1}(2 + -1)$

$= \frac{1}{e}(1)$

$= \frac{1}{e}$ pos conc up

so

$x=-1$ is min

Nov 5-9:31 AM