4.2 The Mean Value Theorem

Objectives:

- I know and understand the mean value theorem

- I can find a value that satisfies the mean value theorem

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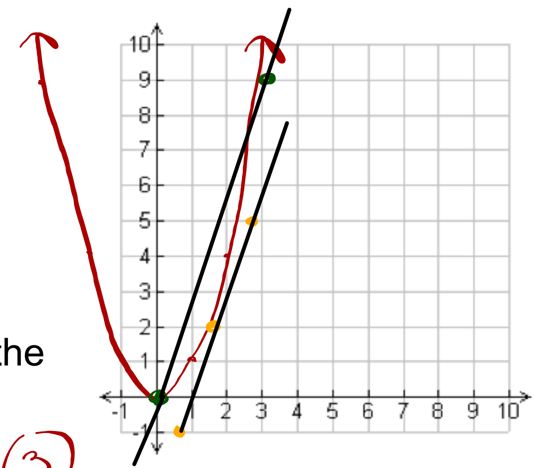
1. Graph $f(x)=x^2$



2. Graph the secant line over [0,3]

3. Find the slope of the secant line.

$\frac{f(3)-f(0)}{3-0} = \frac{9}{3} = 3$

4. Is there an x-value on the interval where the derivative is equal to the slope of the secant line?

$f'(x) = 2x$ $\quad 2x = 3 \quad x = \frac{3}{2}$ or 1.5

5. Draw on the graph the point where this occurs and draw the tangent line.

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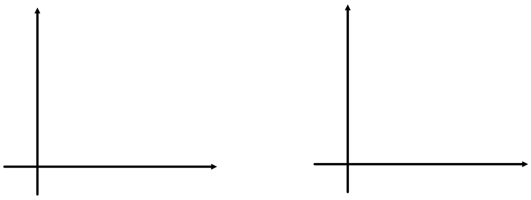
The Mean Value Theorem



Write the MVT in your own words:

slope of sec line = slope of tan line over int [a,b]

What would cause the MVT to fail?

hole,
vert tan line

not differentiable
not continuous

MVT: If $f(x)$
1) is cont. on the int. $[a,b]$
2) is differentiable on int (a,b)

Then there is a number "c" such that $a < c < b$ and

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

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Examples:

a) Verify that the mean value theorem applies to each problem.

b) Find c in (a,b) that satisfies the mean value theorem.

1. $f(x)=x^2$, $[-1,2]$

$f'(x) = 2x$

$f'(c) = 2c$

$$2c = \frac{f(2)-f(-1)}{2--1}$$

average secant

$$2c = \frac{4-1}{3}$$

$2c = 1$

$c = \frac{1}{2}$ or $.5$

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2. $y = e^x$, $[0,2]$

$y' = e^x$

$y'(c) = e^c$

$e^c = \dfrac{f(2) - f(0)}{2 - 0}$

$\ln e^c = \ln\left(\dfrac{e^2 - 1}{2}\right)$

$c = \ln\left(\dfrac{e^2 - 1}{2}\right)$

$c \approx 1.16$

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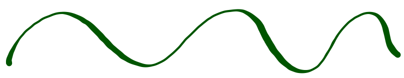
3. $f(x) = \sin x$, $\left[0, \dfrac{\pi}{2}\right]$



$f'(x) = \cos x$

$\cos c = \dfrac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0}$

$\cos c = \dfrac{1 - 0}{\pi/2}$

$\cos^{-1} \cancel{\cos}\, c = \cos^{-1} \dfrac{2}{\pi}$

$c = \cos^{-1}\left(\dfrac{2}{\pi}\right)$

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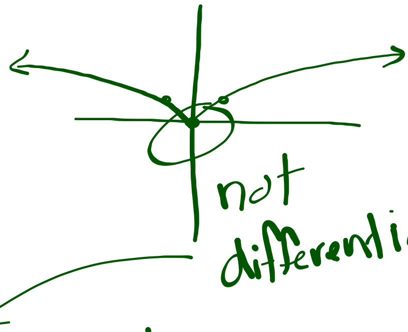
4. $y = x^{\frac{2}{3}}$, $[-1,1]$

$y = \sqrt[3]{x^2}$



not differentiable

MVT Does not apply!

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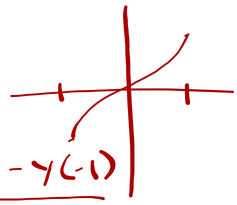
Pg. 202 #5 $y = \sin^{-1} x$, $[-1,1]$



$y' = \frac{1}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{1-c^2}} = \frac{y(1) - y(-1)}{1 - -1}$

$\frac{1}{\sqrt{1-c^2}} = \frac{\frac{\pi}{2} + \frac{\pi}{2}}{2}$

$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2}$

$\left(\sqrt{1-c^2}\right)^2 = \left(\frac{2}{\pi}\right)^2$

$\cancel{1} - c^2 = \frac{4}{\pi^2} - 1$

$\sqrt{c^2} = \sqrt{-\frac{4}{\pi^2} + 1}$

$c = \sqrt{1 - \frac{4}{\pi^2}}$

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Pg 202. #9 $y=\frac{1}{2x^2}$, $[1,3]$

$y = \frac{1}{2}x^{-2}$

$y' = -x^{-3}$

$y' = -\frac{1}{x^3}$

$-\frac{1}{c^3} = \frac{f(3)-f(1)}{3-1}$

$-\frac{1}{c^3} = \frac{1/18 - \frac{1}{2}}{2}$

$-\frac{1}{c^3} = \frac{-4/9}{2}$

$-\frac{1}{c^3} = \frac{-2}{9}$

$\frac{1}{c^3} = \frac{2}{9}$

$c^3 = \frac{9}{2}$

$c = \sqrt[3]{9/2}$

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Oct 16-1:07 PM