3.8 Derivatives of Inverse Functions

Objectives:

- I can find the derivative of an inverse function

- I can find the derivative of an inverse trig function

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Part I:

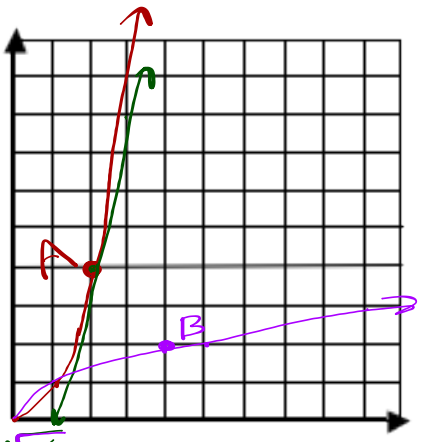
a. Graph $f(x) = x^2 \; for \; x \geq 0$

b. What is f(2)? $f(2) = 4$

c. Draw the point on the graph that you just found. Label it A.

d. What is f '(2)? $= 4$

e. Draw the tangent line at x=2 (point a)



f. Find $f^{-1}(x)$ $x^2 = y$, $\sqrt{y} = \sqrt{x}$ $\rightarrow y = \sqrt{x}$

g. Graph $f^{-1}(x)$

h. Draw the point on $f^{-1}(x)$ that is the reflection of point A. Label it point B.

i. Find the slope of the tangent line of $f^{-1}(x)$ at point B. $\frac{1}{4}$

j. Draw the line.

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k. What is the relationship between the slopes of the tangent lines of this "reflected pair" A and B?

reciprocal (no sign change)

l. Do you think that relationship is true of all "reflected pairs" of points?

Yeppers

Summary:

$f(a)=b \longrightarrow f^{-1}(b)=a$

$f'(a)=c \longrightarrow \left(f^{-1}\right)'(b)=\frac{1}{c}$

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Given $f(x)=x^4-3x^2+4x+2$ find the following:

$f'(x)=4x^3-6x+4$

a) f(1), f '(1)

$f(1)=1-3+4+2=4$

$f'(1)=4-6+4=2$

b) $f^{-1}(4), \left(f^{-1}\right)'(4)$

$f^{-1}(4)=1$

$\left(f^{-1}\right)'(4)=\frac{1}{2}$

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Given $f(x) = \cos x + x$ find the following:

$f'(x) = -\sin x + 1$

a) $f(0), f'(0)$

$f(0) = 1 + 0 = 1$

$f'(0) = 1$

b) $f^{-1}(1), \left(f^{-1}\right)'(1)$

$f^{-1}(1) = 0$

$(f^{-1})'(1) = 1$

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Part II: Inverse trig functions

Find $\frac{dy}{dx}$ if $y = \sin^{-1} x$

$\sin y = \sin \sin^{-1} x$

$\sin y = x$

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y}$

$\sin^2 y = x^2$

$= \frac{1}{\sqrt{1 - x^2}}$

Find $\frac{dy}{dx}$ if $y = \tan^{-1} x$

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Rule Sheet: 67-72

67. $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$ 68. $\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$

69. $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$ 70. $\frac{d}{dx}(\cot^{-1}x)=\frac{-1}{1+x^2}$

71. $\frac{d}{dx}(\sec^{-1}x)=\frac{1}{|x|\sqrt{x^2-1}}$

72. $\frac{d}{dx}(\csc^{-1}x)=\frac{-1}{|x|\sqrt{x^2-1}}$

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Helpful identities

$$\sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right)$$

$$\csc^{-1}x=\sin^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}x=\tan^{-1}\left(\frac{1}{x}\right)$$

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Derive:

$f(x) = \cos^{-1}(3x)$

$f'(x) = \frac{-1}{\sqrt{1-(3x)^2}} \cdot 3$

$f'(x) = \frac{-3}{\sqrt{1-9x^2}}$

$y = \cos^{-1}\left(\frac{3}{x}\right)$

$y = \sec^{-1}\left(\frac{x}{3}\right)$

$y' = \frac{1}{\left|\frac{x}{3}\right|\sqrt{\left(\frac{x}{3}\right)^2-1}} \cdot \frac{1}{3}$

$y' = \frac{1}{3\left|\frac{x}{3}\right|\sqrt{\frac{x^2}{9}-1}}$

$g(x) = \sin^{-1} x^2$

$g'(x) = \frac{1}{\sqrt{1-x^4}} \cdot 2x$

$g'(x) = \frac{2x}{\sqrt{1-x^4}}$

$g(x) = \cos^{-1}\left(\frac{4}{x^2}\right)$

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Derive:

$f(x) = \csc^{-1}(3x+2)$

$(x^2+2)^{\frac{1}{2}}$

$\frac{1}{2}(x^2+2)^{-\frac{1}{2}} \cdot 2x$

$\frac{2x}{2\sqrt{x^2+2}}$

$y = \tan^{-1}\sqrt{x^2+2}$

$y' = \frac{1}{1+(\sqrt{x^2+2})^2} \cdot \frac{x}{\sqrt{x^2+2}}$

$y' = \frac{x}{(x^2+3)\sqrt{x^2+2}}$

$y = \sec^{-1}(3x^2)$

$f(x) = \sin^{-1}\left(\frac{1}{x}\right)$

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