3.7 Implicit Differentiation

Objectives:

- I can take the derivative by implicit differentiation

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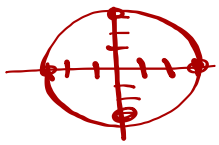
$(x-h)^2+(y-k)^2=r^2$



Consider $x^2+y^2=9$

$2x\frac{dx}{dx}+2y\frac{dy}{dx}=0$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{2y}$

$\frac{dy}{dx}=-\frac{x}{y}$

1. Find the equation of the tangent line through $(2,-\sqrt{5})$

$m=\frac{2}{\sqrt{5}}$

$y+\sqrt{5}=\frac{2}{\sqrt{5}}(x-2)$

$y=\frac{2}{\sqrt{5}}(x-2)-\sqrt{5}$

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Steps for implicit differentiation

I. Take the derivative of every term with respect to x

II. Solve for $\frac{dy}{dx}$

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For each of the following: A) find $\frac{dy}{dx}$

B) Find the equation of the tangent and normal line at the given point

$y^2 = x$ (4, -2)

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$m = \frac{1}{2(-2)} = -\frac{1}{4}$

tan line

$y = -\frac{1}{4}(x-4) - 2$

norm line

$y = 4(x-4) - 2$

$x^2 - y^2 = 25$ (-5,0)

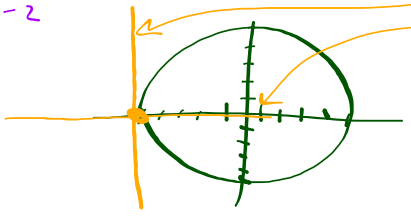
$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

$m = \frac{-5}{0}$ undefined so $x = -5$

$y = 0$



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Find $\frac{dy}{dx}$ $x^2 = \frac{x^2+1}{y^2}$

$(y^4)\,2x = \frac{y^2(2x) - (x^2+1)\left(2y\frac{dy}{dx}\right)}{y^4} \cdot y^4$

$2xy^4 = 2xy^2 - \left(2x^2y\frac{dy}{dx} + 2y\frac{dy}{dx}\right)$

$2xy^4 = 2xy^2 - 2x^2y\frac{dy}{dx} - 2y\frac{dy}{dx}$

$2xy^4 - 2xy^2 = -2x^2y\frac{dy}{dx} - 2y\frac{dy}{dx}$

$\frac{2xy^4 - 2xy^2}{-2x^2y - 2y} = \frac{\frac{dy}{dx}\left(-2x^2y - 2y\right)}{(-2x^2y - 2y)}$

$\frac{2xy^2(y^2-1)}{-2y(x^2+1)} = \frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{xy(y^2-1)}{x^2+1}$

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Find $\frac{dy}{dx}$ $y^2 = \frac{x^2-1}{x^3}$

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Find $\frac{dy}{dx}$ $x^2+2xy+y^2=0$

Prod rule

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x-2y$

$\frac{dy}{dx}(2x+2y) = -2x-2y$

$\frac{dy}{dx} = \frac{-2x-2y}{2x+2y} = \frac{-2(x+y)}{2(x+y)} = -1$

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Find $\frac{d^2y}{dx^2}$ $y^2=x^2+2x$

$2y\frac{dy}{dx} = 2x+2$

$\frac{dy}{dx} = \frac{2(x+1)}{2y}$

$\frac{dy}{dx} = \frac{x+1}{y}$

$\frac{d^2y}{dx^2} = \frac{y(1)-(x+1)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{y^2}{y}-\frac{(x+1)^2}{y}}{y^2} = \frac{\frac{y^2-(x+1)^2}{y}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y^2-(x+1)^2}{y}\cdot\frac{1}{y^2} = \frac{y^2-(x+1)^2}{y^3}$

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