3.3 Rules for Differentiation

Objectives:

-I can use the rules of differentiation to find the derivative of a function

- I can find the second derivative of a function

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Rule Sheet Numbers 1-4

$$1.\ \frac{d}{dx}(c)=0$$

$$3.\ \frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$$

Power Rule! ☺

$$2.\ \frac{d}{dx}(cu)=c\frac{du}{dx}$$

$$4.\ \frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$$

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Find each derivative:

1. $y = 2x + 4$

$y' = 2 + 0$

$y' = 2$

2. $f(x) = x^6$

$f'(x) = 6x^5$

3. $y = 2x^3$

$y' = 2 \cdot 3x^2$

$y' = 6x^2$

4. $y = 2\pi^5$ (constant)

$y' = 0$

5. $g(x) = \frac{x^2}{3}$

$g(x) = \frac{1}{3}x^2$

$g'(x) = 2 \cdot \frac{1}{3}x = \frac{2}{3}x$

6. $f(x) = \frac{1}{x^5}$

$f(x) = x^{-5}$

$f'(x) = -5x^{-6}$

$= \frac{-5}{x^6}$

7. $y = \frac{4}{x}$

$y = 4x^{-1}$

$y' = -4x^{-2}$

$y' = \frac{-4}{x^2}$

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8. $f(x) = 2x^2 - 5x + 1$

$f'(x) = 4x - 5 + 0$

$f'(x) = 4x - 5$

9. $y = \frac{3}{x^2} + \frac{4}{x^3} - \frac{8}{x^4}$

$y = 3x^{-2} + 4x^{-3} - 8x^{-4}$

$y' = -6x^{-3} - 12x^{-4} + 32x^{-5}$

$y' = \frac{-6}{x^3} - \frac{12}{x^4} + \frac{32}{x^5}$

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Rule Sheet The Product Rule

$$5.\ \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$ or $u \cdot v' + v \cdot u'$

$u' = 2x$
$v' = 1$

10. $y = (x^2+3)(x-2)$

$y' = (x^2+3)(1) + (x-2)(2x)$

$y' = (x^2+3) + 2x^2 - 4x$

$y' = 3x^2 - 4x + 3$

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Rule Sheet
The Quotient Rule

$$6.\ \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ or $\frac{v \cdot u' - u \cdot v'}{v^2}$

11. $y = \frac{4x-2}{x+1}$

$y' = \frac{(x+1)(4) - (4x-2)(1)}{(x+1)^2}$

$y' = \frac{4x+4-4x+2}{(x+1)^2}$

$y' = \frac{6}{(x+1)^2}$

12. $g(x) = \frac{2x}{x^2+1}$

$g'(x) = \frac{(x^2+1)(2) - (2x)(2x)}{(x^2+1)^2}$

$g'(x) = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$g'(x) = \frac{-2x^2+2}{(x^2+1)^2} = \frac{-2(x^2-1)}{(x^2+1)^2}$

13. $h(x) = \frac{6x^2-7x}{x}$

15. $f(x) = \frac{8}{4+x^2}$

$y' = \frac{(x^3)(0) - (e^2)(3x^2)}{(x^3)^2}$

$y' = \frac{0 - 3e^2x^2}{x^6}$

$y' = \frac{-3e^2}{x^4}$

14. $y = \frac{e^2}{x^3} = e^2x^{-3}$

$y' = -3e^2x^{-4}$

$y' = \frac{-3e^2}{x^4}$

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Tips for finding derivatives

- Know the rules and stick to them

- Don't jump right in the rule. It might be easier to simplify first

- Practice, practice, practice! You will want to be able to find derivates quickly and accurately

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The Second Derivative

Notation: $f''(x)$ or $\frac{d^2y}{dx^2}$

16. Find the second derivative of $y = x^4 - 7x^3 + 5x^2 - 4x + 10$

$y' = 4x^3 - 21x^2 + 10x - 4$

$y'' = 12x^2 - 42x + 10$

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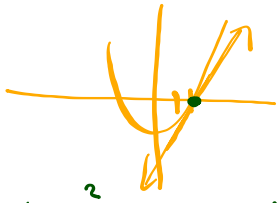
17. Find the equation of the tangent line to $f(x)=x^2-4x+1$ at x=3



$f'(x) = 2x-4$

$f'(3) = 2(3)-4 = \boxed{2}$

$f(3) = (3)^2-4(3)+1$

$= -2$ $(3,-2)$

or $y+2 = 2(x-3)$

or $y = 2(x-3)-2$

or $y = 2x-8$

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deriv = 0

Find the horizontal tangents of the curve.

$$y = 4x^3 - 6x^2 - 1$$

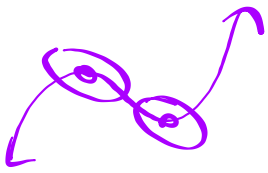
$y' = 12x^2 - 12x$

$12x^2 - 12x = 0$

$12x(x-1) = 0$

$12x = 0$ $x-1=0$

$x = 0$ $x = 1$



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18. In the formula $V = \frac{4r+s^2}{t} + 16rst^2$ r and s are constants. Find $\frac{dV}{dt}$

$\frac{dV}{dt} = \frac{t(0) - (4r+s^2)(1)}{t^2} + 32rst$

$= \frac{-4r-s^2}{t^2} + 32rst$

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Suppose u and v are functions of x that are differentiable at x=2 and that u(2)=3, u'(2)= -4, v(2)=1, and v'(2)=2. Find the values of the following derivatives at x=2

a. $\frac{d}{dx}(uv)$ Prod Rule

$u \cdot v' + v \cdot u'$

$(3)(2) + (1)(-4)$

$6 + -4 = 2$

b. $\frac{d}{dx}\left(\frac{u}{v}\right)$

c. $\frac{d}{dx}\left(\frac{v}{u}\right)$

d. $\frac{d}{dx}(3u - 2v + 2uv)$

$3u' - 2v' + (2u \cdot v' + v \cdot 2u')$

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