3.2 Differentiability

Objectives:

- I can determine if a function is differentiable

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Given $f(x)=\frac{1}{2}x^2$ find the slope of the secant line of $f(x)$ over [1,5]

$$\frac{f(5)-f(1)}{5-1} = \frac{12.5 - .5}{4} = \frac{12}{4} = \boxed{3}$$

Find the average rate of change of f(x) over [2,4]

$$\frac{f(4)-f(2)}{4-2} = \frac{8-2}{2} = \boxed{3}$$

Find the average rate of change of f(x) over [2.5,3.5]

$$\frac{f(3.5)-f(2.5)}{3.5-2.5} = \boxed{3}$$

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Choose an interval from the last slide that gives the best approximation of f'(3). Explain why your method gives an accurate approximation

$[3-h, 3+h]$

Could you use this process to find f'(x)? Why or why not?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{(x+h)-(x-h)}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h}$$

$f(x) = \frac{1}{2}x^2$

Find f'(3)

$$\lim_{h\to 0} \frac{\frac{1}{2}(3+h)^2 - \frac{1}{2}(3-h)^2}{2h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}(9+6h+h^2) - \frac{1}{2}(9-6h+h^2)}{2h}$$

$$= \lim_{h\to 0} \frac{\frac{9}{2} + 3h + \frac{1}{2}h^2 - \frac{9}{2} + 3h - \frac{1}{2}h^2}{2h} = \lim_{h\to 0} \frac{6h}{2h} = 3$$

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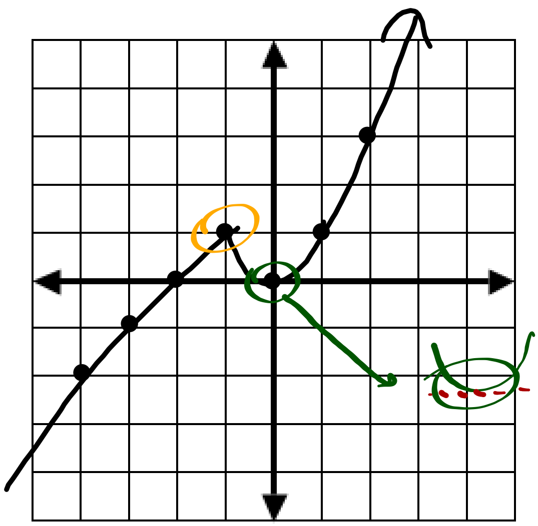
Differentiability

$$f(x) = \begin{cases} x+2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$$



The graph of f(x) is given. What do you observe at x= -1 and x=0?

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Definitions

LH Deriv = RH Deriv

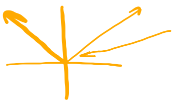
☆f(x) is differentiable at x if f'(x) exists

☆f(x) is a differentiable function if f'(x) exists for all x in the domain

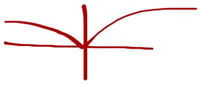
In what cases would a function not be differentiable at a point x=a?

no tan line

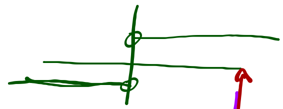
① Corner → $y=|x|$



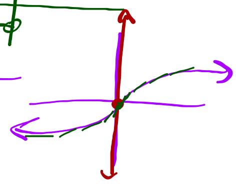
② Cusp → $y^{\frac{2}{3}}$



③ Discontinuity piecewise



④ Vertical Tan Line $y=\sqrt[3]{x}$



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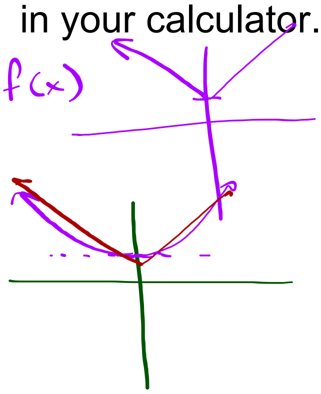
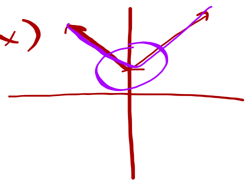
Exploring differentiability with local linearity

Compare $f(x)=\sqrt{x^2+.001}+.99$ and $g(x)=|x|+1$ in your calculator.

f(x)



g(x)



How do differentiability and continuity relate to each other?

* If a func. is differentiable ⇒ continuous

* If a func. is continuous ⇏ differentiable

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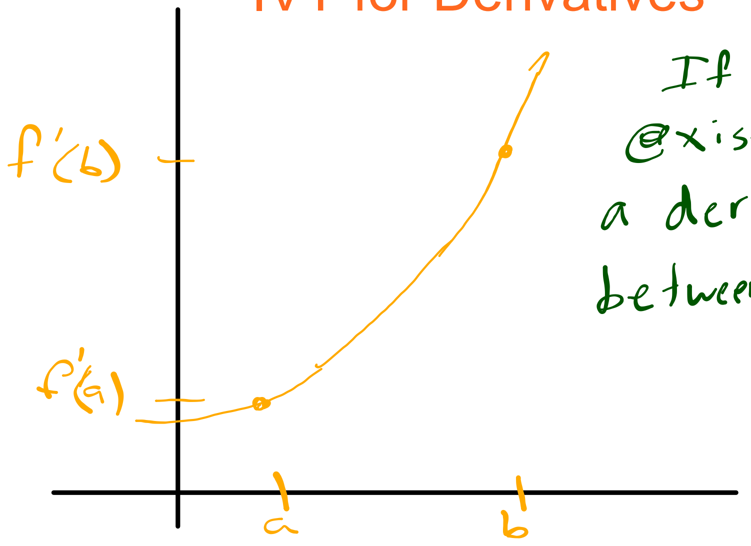
IVT for Derivatives



If f'(a) and f'(b) exist there exists a deriv. everywhere between a and b.

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