3.1 The Derivative

Objectives:

- I can use the definition of a derivative to determine the derivative of a function

- I can graph the derivative of a function

Aug 7-11:29 AM

Review

What is the difference quotient theorem?

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Instantaneous R.O.C., Slope of tangent line,

List at least 2 things that the difference quotient theorem tells us

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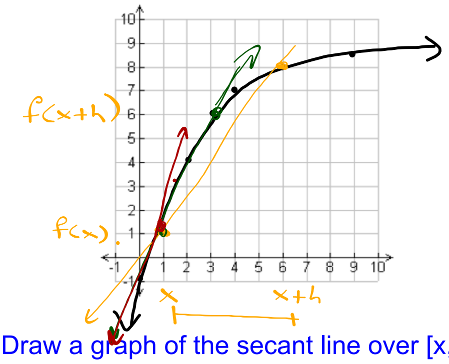
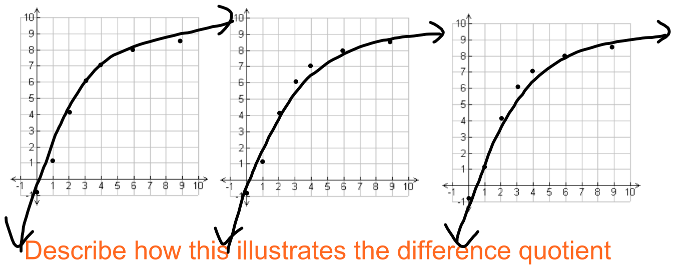
Given the graph below, label f(x) and f(x+h) on the y-axis



Draw a graph of the secant line over [x, x+h] above and express the slope in terms of x and h

Repeat the process each time using a smaller h



Describe how this illustrates the difference quotient theorem

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Find the instantaneous rate of change of $f(x)=x^2$ at $x=3.5$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(3.5+h)-f(3.5)}{h}$$

$$=\lim_{h\to 0}\frac{(3.5+h)^2-12.25}{h}=\lim_{h\to 0}\frac{\cancel{12.25}+7\cancel{h}+h^{\cancel{2}}\cancel{-12.25}}{\cancel{h}}$$

$$=\lim_{h\to 0}7+h=7+0=7$$

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Definition: When it exists, the difference quotient theorem is called the Derivative

Rule Sheet #24: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Notation: Different ways of writing the derivative (P. 101)

$f'(x)$ y' $\frac{dy}{dx}$ $\frac{d}{dx}f(x)$

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Use the definition of the derivative to find the derivative of each function

$f(x)=x^2$ $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0}\frac{\cancel{x^2}+2x\cancel{h}+h^2-\cancel{x^2}}{\cancel{h}} = \lim_{h\to 0} 2x+\cancel{h} = 2x$

What is the significance of your answer? How are the process and result different from the warm up exercise?

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Use the definition of the derivative to find the derivative of each function

$$f(x) = 2x$$

$$y = \frac{1}{x} \qquad \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} = \lim_{h\to 0} \frac{1}{h}\cdot\left(\frac{1x}{x(x+h)} - \frac{1(x+h)}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{\cancel{x}-\cancel{x}-\cancel{h}}{x(x+h)}\right) = \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

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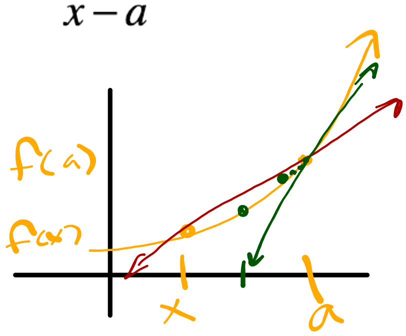
There are several definitions of the derivative. Explain or show why $f'(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ is a definition of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



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Use the second definition of the derivative to find y'

$$y=x^4$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$(x+h)(x+h)(x+h)(x+h)$$

$$= \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{x^4-a^4}{x-a}$$

$$= \lim_{x\to a} \frac{(x^2)^2-(a^2)^2}{x-a} = \lim_{x\to a} \frac{(x^2-a^2)(x^2+a^2)}{x-a}$$

$$= \lim_{x\to a} \frac{\cancel{(x-a)}(x+a)(x^2+a^2)}{\cancel{x-a}} = (2a)(2a^2)$$

$$= 4a^3$$

Why would the second definition be better than the first in this case?

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Think of another function where using the second definition would be better? Try to think of one that is not a polynomial.

Why would the second definition of the derivative be better for your function?

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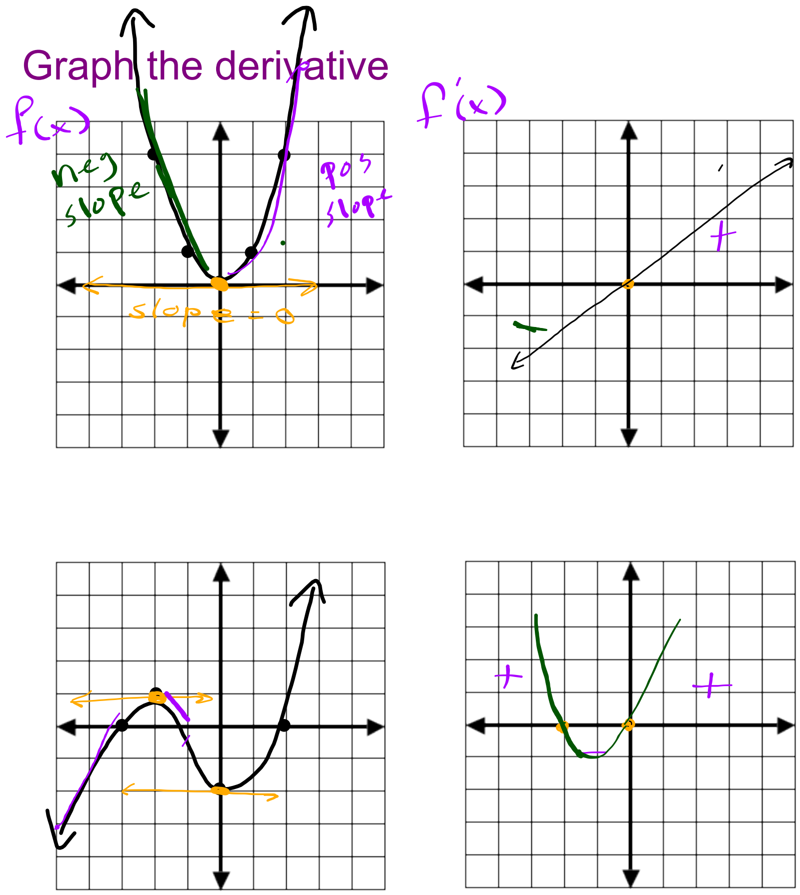
Graph the derivative



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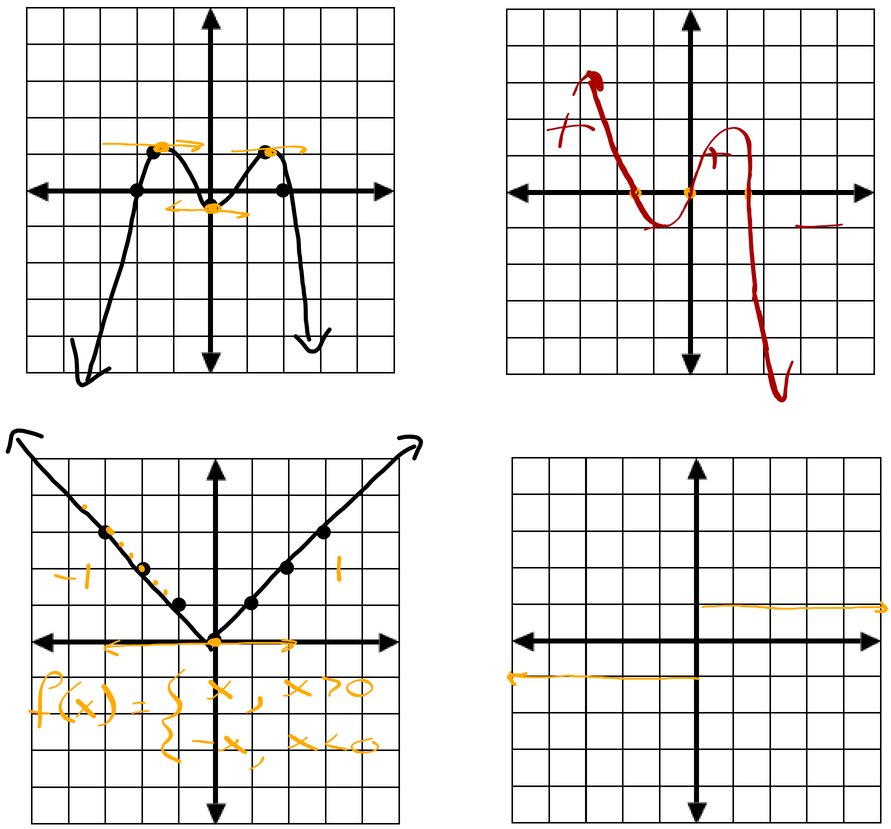

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