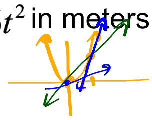
Calculus 2.4 Rates of Change

The position of a rock dropping on Mars is represented by the equation $f(t) = 1.86t^2$ in meters when t is in seconds.



Δ change

a) Find the average speed of the rock for the first two seconds.

$$\frac{\Delta y}{\Delta t} = \frac{f(2)-f(0)}{2-0} = \frac{7.44-0}{2} = 3.72 \text{ m/s}$$

b) Find the average speed over [1,4].

$$\frac{f(4)-f(1)}{4-1} = \frac{29.76-1.86}{3} = 9.3 \text{ m/s}$$

c) Find the instantaneous speed at 3 sec.

$$\frac{f(3.1)-f(3)}{3.1-3} = \frac{17.875-16.74}{0.1} = 11.346 \text{ m/s}$$

$$\frac{f(3.01)-f(3)}{3.01-3} = \frac{16.852-16.74}{0.01} = 11.179 \text{ m/s}$$

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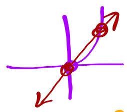
Average rate of change-

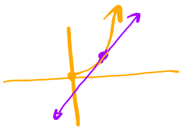


slope of secant line

secant line goes through 2 pts on function

$[a,b]$ $\frac{f(b)-f(a)}{b-a}$

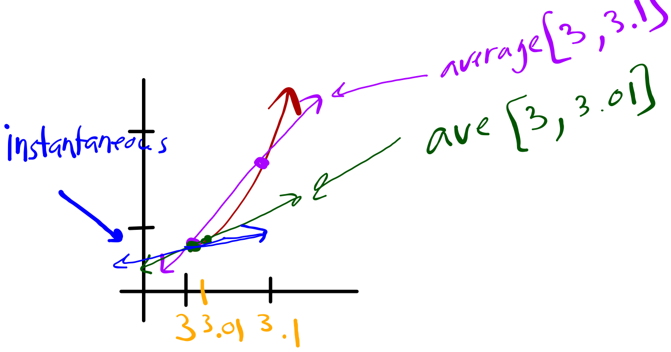
Instantaneous rate of change-



slope of tangent line

tangent line only touches function once

Draw a general sketch how we calculated the instantaneous rate of change.



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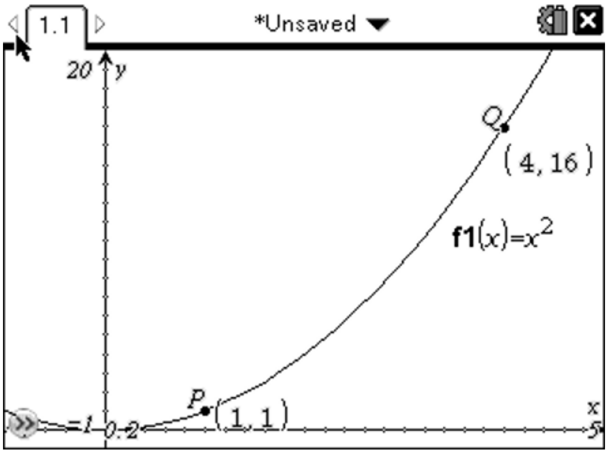
What happens as h approaches zero.



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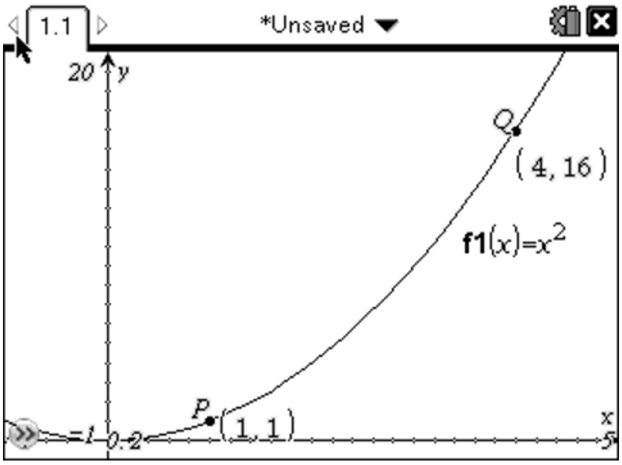
Estimate the instantaneous velocity at t=1



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Difference Quotient Theorem- Instantaneous R.O.C.

Slope of the tangent line at x=a is:

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

*Slope will be different for different x-values

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For each of the following:

a) Find the average rate of change of the given interval.
b) Find the instantaneous rate of change at a.
c) Write an equation of the tangent line.
d) Write an equation of the normal line. (perpendicular to the tangent line).

1. $f(x)=\frac{1}{4}x^2$ over [1,3], a=1 $(1+h)$ $f(1+h)=\frac{1}{4}(1+h)^2$

a) $\frac{f(3)-f(1)}{3-1}=\frac{\frac{9}{4}-\frac{1}{4}}{2}=\frac{2}{2}=\boxed{1}$

b) $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$: $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\frac{\cancel{\frac{1}{4}}+\frac{1}{2}\cancel{h}+\frac{1}{4}h^{\cancel{2}}-\cancel{\frac{1}{4}}}{\cancel{h}}=\lim_{h\to 0}\frac{1}{2}+\frac{1}{4}h=\frac{1}{2}$

c) $m=\frac{1}{2}$
pt. $(1,\frac{1}{4})$
$y-\frac{1}{4}=\frac{1}{2}(x-1)$

d) $y-\frac{1}{4}=-2(x-1)$

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2. $f(x)=\frac{1}{x}$ over [-2,-1], a=2

a) $\frac{f(-1)-f(-2)}{-1--2}=\frac{-1+\frac{1}{2}}{1}=\boxed{-\frac{1}{2}}$

b) $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{1}{a+h}-\frac{1}{a}}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{1a}{(a+h)a}-\frac{1(a+h)}{a(a+h)}\right)$

$\lim_{h\to 0}\frac{1}{\cancel{h}}\left(\frac{\cancel{a}-\cancel{a}-\cancel{h}}{a(a+h)}\right)=\lim_{h\to 0}\frac{-1}{a(a+h)}=\frac{-1}{a(a+0)}$

$=\frac{-1}{a^2}$ so $-\frac{1}{2^2}=\boxed{-\frac{1}{4}}$

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3. $f(x)=x^2-x$ over [0,4], a=3 $\quad a^2+2ah+h^2-(a+h)$

a) $\frac{f(4)-f(0)}{4-0}=\frac{12-0}{4}=\boxed{3}$

b) $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\cancel{a^2}+2a\cancel{h}+h^{\cancel{2}}-\cancel{a}-\cancel{h}^{-1}-\cancel{a^2}+\cancel{a}}{\cancel{h}}$

$=\lim_{h\to 0}2a+h-1=2a-1$ so $2(3)-1=\boxed{5}$

c) pt. (0,0) so $y=5x$
$m=5$

d) $y=-\frac{1}{5}x$

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4. Find the slope of the tangent line of

$f(x) = -2x^2 + 1$ at x=a.

$f(a) = -2a^2+1$ $f(x+h) = -2(x+h)^2+1$
$= -2(x^2+2hx+h^2)+1$
$= -2x^2 - 4hx - 2h^2 + 1$

$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{-2x^2 - 4hx - 2h^2 + 1 - (-2a^2+1)}{h}$

$= \lim_{h\to 0} \frac{\cancel{-2x^2} - 4\cancel{h}x - 2h^{\cancel{2}} \cancel{+1} + \cancel{2x^2} \cancel{-1}}{\cancel{h}} = \lim_{h\to 0} -4x - 2h^{\nearrow 0}$

$= -4x$ and $x=a$ so $\boxed{-4a}$

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5. Find the slope of the tangent line of

$f(x) = 9 - 3x^2$ at x=a.

Book Example pg. 92 #8 a) and b)...Lunar Data

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