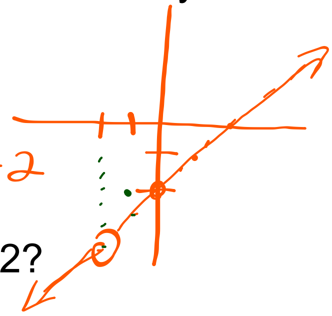
2.1 Limits

Consider $f(x)=\frac{x^2-4}{x+2}$. What observations can you make?

$\frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}}$ $\qquad y=x-2$

hole at $x=-2$



How can we describe what happens at x= -2?

as x approaches -2, y approaches -4

So $\lim_{x\to -2} f(x) = -4$

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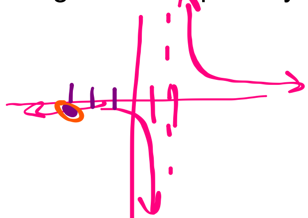
Find the Limit: $\lim_{x\to -3}\frac{x+3}{x^2+x-6} = \lim_{x\to -3}\frac{\cancel{x+3}}{\cancel{(x+3)}(x-2)} = \lim_{x\to -3}\frac{1}{x-2}$

Finding limits Analytically (Algebraically)

* Simplify

$\lim_{x\to -3}\frac{1}{x-2} = \frac{1}{-3-2} = \boxed{-\frac{1}{5}}$

Finding limits Graphically:



Finding limits Numerically:

look at values to left and right

Estimate $\qquad -0.2$

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Find each limit.

$$1.\lim_{x\to 2}\left(x^2+3x-5\right)=$$

$= 2^2 + 3(2) - 5$

$= 4 + 6 - 5$

$= \boxed{5}$

$$2.\lim_{x\to \pi}(\sin x\cos x)=$$

$= \sin\pi \cos\pi$

$= (0)(-1)$

$= 0$

$$3.\lim_{x\to 4}\frac{x-4}{x^2-16}=$$

$$= \lim_{x\to 4}\frac{\cancel{x-4}}{(\cancel{x-4})(x+4)} = \lim_{x\to 4}\frac{1}{x+4} = \frac{1}{8}$$

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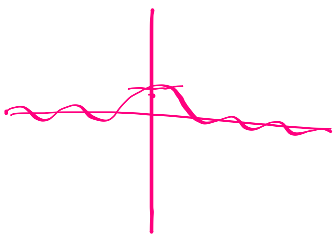
Find the limit.

$$4.\lim_{x\to 0}\frac{\sin x}{x}=$$

$\frac{\sin 0}{0} = \frac{0}{0}$



$= \boxed{1}$

$$5.\lim_{x\to 0}\frac{\sin x+x}{x}= \lim_{x\to 0}\frac{\sin x}{x}+\frac{x}{x}$$

$\lim_{x\to 0} 1$

$$= \lim_{x\to 0}\frac{\sin x}{x} + \lim_{x\to 0}\frac{x}{x}$$

$= 1 + 1$

$= 2$

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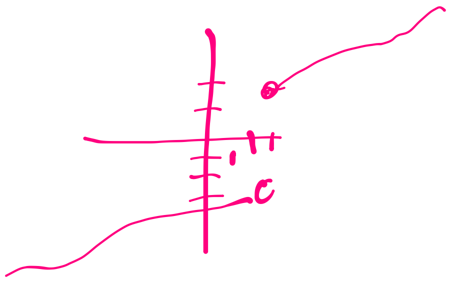
Left and right hand limits.



$$\lim_{x \to c^+} f(x) \qquad \lim_{x \to c^-} f(x)$$

RH LH

Definition of a limit:

"if and only if" RH = LH

$$\lim_{x \to c} f(x) = \text{L} \quad \textbf{iff} \quad \lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = \text{L}$$

the limit doesn't necessarily equal the function!!!

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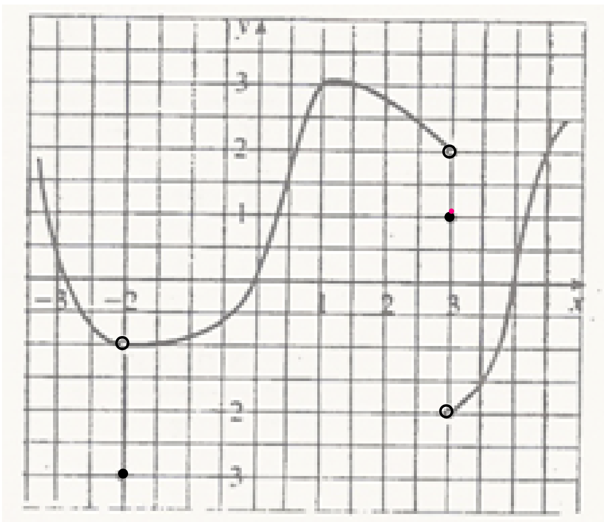

(a) $\lim_{x \to 1} f(x) = 3$ (b) $\lim_{x \to 3^-} f(x) = 2$ (c) $\lim_{x \to 3^+} f(x) = -2$

(d) $\lim_{x \to 3} f(x) =$ DNE (e) $f(3) = 1$ (f) $\lim_{x \to -2^-} f(x) = -1$

(g) $\lim_{x \to -2^+} f(x) = -1$ (h) $\lim_{x \to -2} f(x) = -1$ (i) $f(-2) = -3$

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Properties of Limits:

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$$1. \lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$2. \lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

Given that $\lim_{x\to a} f(x) = -3,\ \lim_{x\to a} g(x) = 0,\ \lim_{x\to a} h(x) = 8,$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to a}\left[f(x)+h(x)\right] = -3+8$ $= -5$

(b) $\lim_{x\to a}\left[f(x)\right]^2 = (-3)^2 = 9$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} =$

(d) $\lim_{x\to a}\dfrac{1}{f(x)} =$

(e) $\lim_{x\to a}\dfrac{f(x)}{h(x)} =$

(f) $\lim_{x\to a}\dfrac{g(x)}{f(x)} =$

(g) $\lim_{x\to a}\dfrac{f(x)}{g(x)} =$

(h) $\lim_{x\to a}\dfrac{2f(x)}{h(x)-f(x)} =$

$$\frac{2(-3)}{8-(-3)} = \frac{-6}{11}$$

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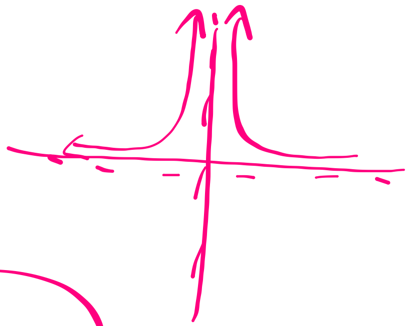
16) $\lim_{x\to 0} \dfrac{1}{x^2}$



$= \infty$

Aug 23-6:53 AM