CHAPTER 9 - COUNTING PRINCIPLES AND PROBABILITY

Probability is the ____

Probability is used in many real-world fields, such as insurance, medical research, law enforcement, and political science.

SECTION 9-1 INTRODUCTION TO PROBABILITY

Objectives:

Find the theoretical probability of an event.
Apply the Fundamental Counting Principle.

How do some businesses, such as life insurance companies and gambling establishments, make dependable profits on events that seem unpredictable? The answer is that the overall likelihood, or **probability**, of an event can be discovered by observing the results of a large number of repetitions of the situation in which the event may occur.

The terminology used for probability is given below. The sample is the rolling of a number cube.

DEFINITION	EXAMPLE
Trial: A systematic opportunity for an event to occur	rolling a number cube
Experiment: one or more trials	rolling a number cube 10 times
Sample space: the set of all possible outcomes of an event	1, 2, 3, 4, 5, 6
Event: an individual outcome **or** any specified combination of outcomes	rolling a 3 rolling a 3 *or* rolling a 5

Probability is expressed as a number from 0 to 1. It is often written as a fraction, decimal, or percent.

Experimental probability is a ____

____

Theoretical probability is based ____

Outcomes are **random** if ____

Theoretical Probability

If all outcomes in a sample space are equally likely, then the theoretical probability of event A, denoted P(A), is defined by

$$P(A) = \frac{\text{number of outcomes in event } A}{\text{number of outcomes in sample space}}$$

Ex. 1 A bag contains 2 white marbles, 4 red marbles, and 10 green marbles. What is the probability of drawing a red marble?, a blue marble? a white marble or a red marble out of the bag?

Remember that probability is between $0 \leq P(A) \leq 1$.

- An impossible event has a probability of 0.
- An event that must occur has a probability of 1.
- The sum of the probabilities of all outcomes in a sample space is 1.

Ex. 2 A bag contains 10 red, 5 black, 4 yellow and 2 blue jellybeans. Find the probability of selecting a red, a black, a yellow, a blue, a purple, a red or black or yellow or blue jellybean.

There are several ways to determine the size of a sample space for an event that is a combination of two or more outcomes. One way is a tree diagram.

Ex. 3 Make a tree diagram for a fast food restaurant that has a hamburger with the choice of Coke or Dr. Pepper for a drink, and a side order of regular fries, crispy fries, or curly fries.

Tree diagrams illustrate the *Fundamental counting Principle.*

Fundamental Counting Principle

If there are *m* ways that one event can occur and *n* ways that another event can occur, then there are $m \times n$ ways that both events can occur.

Ex. 4 In order to purchase a Power Ball Ticket you have to choose 5 numbers 1 through 69 and a "power ball" number that is 1 though 26. How many different tickets can you purchase?

Ex. 5 How many Utah license plates can be made? (3 numbers followed by 3 letters)

The **odds** in favor of an event are defined as the number of ways the event can happen ***a*** compared to the number of ways it can fail ***b***. We write as the ratio ***a:b***

Ex. 8 Find the odds of a team winning if it wins 15 games and loses 5 games

What is the probability of winning if the odds in favor of an event are a:b, or a to b, then the probability of the event is $\frac{a}{a+b}$.

Ex. 9 Find the probability of the event if the given odds in favor of the event is 2 to 7.

SECTION 9-2 Permutations

Objectives:

- Solve problems involving linear permutations of distinct of indistinguishable objects.
- Solve problems involving circular permutations.

order matters

A **permutation** is an arrangement of objects in a specific order. When objects are arranged in a row, the permutation is called a **linear permutation.** Unless otherwise noted, the term *permutation* will be used to mean *linear permutations.*

Ex. 1 Make an organized list of the possible permutation of the letters A, B, and C.

ABC BAC CAB
ACB BCA CBA

In 4 different letters there are 4 × 3 × 2 × 1 , or 24, possible arrangements. You can use *factorial notation* to abbreviate this product: 4! = 4 × 3 × 2 × 1 = 24.

26!

Permutations of *n* Objects
The number of permutations of *n* objects is given by *n*!.

These objects are taken one at a time.

Ex. 2 In 12-tone music, each of the 12 notes in an octave must be used exactly once before any are repeated. A set of 12 tones is called a tone row. **How many different tone rows are possible?**

12! = 479001,600

Note: 1! = 1 and 0! = 1

Ex. 3 Evaluate each expression.

a. $(6-2)!$ **b.** $6!-2!$ **c.** $\dfrac{9!}{(5-2)!}$ **d.** $\dfrac{5!0!}{(7-6)!}$ $= 5! = 120$

$4! = \boxed{24}$ $720 - 2 = \boxed{718}$ $\dfrac{9!}{3!} = 60480$

Permutations of n Objects Taken r at a Time

The number of permutations of n objects taken r at a time, denoted by $P(n,r)$ or ${}_nP_r$ is given by $P(n,r) = \boxed{{}_nP_r = \dfrac{n!}{(n-r)!}}$, where $r \le n$.

Ex. 4 Evaluate

a. ${}_6P_2$ $\dfrac{6!}{(6-2)!} = \dfrac{6!}{4!}$ 30 **b.** ${}_{15}P_{10}$ $\dfrac{15!}{5!} = 1.09 \times 10^{10}$

Ex. 5 Find the number of permutations of the first 8 letters of the alphabet taking 5 letters at a time.

${}_8P_5 = \dfrac{8!}{(8-5)!} = \dfrac{8!}{3!} = 6720$

Ex. 6 In how many ways can 8 new employees be assigned to 11 vacant offices?

Ex. 7 In how many ways can a teacher arrange 6 students in the front row of a classroom with 32 students?

Ex 7

Permutation With Identical Objects

$$\frac{n!}{r_1!r_2!r_3!\ldots r_k!}$$

$8! = 40{,}320$

Ex. 8 Find the number of permutations of the letters in the word **football.**

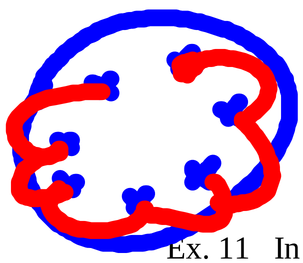
0's $\dfrac{8!}{(2!2!)}$ 1's $= \dfrac{8!}{4} = 10080$

Ex. 9 If I have 2 green, 3 blue, 7 yellow, and 1 red M&M's. If I eat them one by one, how many ways can I eat them.

$\dfrac{13!}{(2!3!7!)} = 102{,}960$

Ex 10 I have 5 black 6 red and 12 white flags in the color guard closet how many different ways can you line up the flags?

$\dfrac{23!}{(5!\,6!\,12!)}$



Circular Permutations
If n distinct objects are arranged around a circle, then there are $(n - 1)!$
Circular permutations of the n *objects.*

Ex. 11 In how many ways can 7 different appetizers by arranged on a circular tray?

$(7-1)! = 6! = 720$

SECTION 9-3 Combinations

Objectives:

•Solve problems involving combinations.
•Solve problems by distinguishing between permutations and combinations

I have an auto dealership on State St. I have 8 different cars on my lot and I need to display them. How many different ways can I do the following?

- I have 8 stalls on State Street
- I have 5 stalls on State Street
- I have an 8 car rotating display at the Expo
- If two are Buicks and three are Rangers and the others are different on State Street
- I have 5 stalls and order doesn't matter

Recall that a permutation is an arrangement of objects in a specific order. An arrangement of objects in which order is not important is called a **combination.**

Combinations of *n* Objects Taken *r* at a Time

The number of combinations of n objects taken r at a time, is given by

$$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}, \text{ where } 0 \le r \le n$$

All of the notations in the above box have the same meaning. All are read as "n choose r." The formula for combinations is like the formula for permutations except that it contains the factor of r!

Ex. 1 Find the value of each expression.

a. ${}_{9}C_{5}$

b. ${}_{12}C_{12}$

c. $\frac{6!}{2!4!} \times \frac{5!}{4!1!}$

d. $\frac{{}_{14}C_{5} \times {}_{9}C_{7}}{{}_{23}C_{12}}$

Ex. 2 Find the number of ways to purchase 3 different kinds of juice from a selection of 10 different juices.

Ex. 3 Find the number of ways in which the committee can select:

a. 4 people from a group of 7.

b. 3 people from a group of 10.

Ex. 4 How many different ways can I make a 9 person batting order from a team of 12 players?

Ex. 5 Which is larger, ${}_{10}C_{7}$ or ${}_{10}P_{7}$? Does a given number of objects have more combinations or permutations?

Ex. 6 A pizza parlor offers a selection of 3 different cheeses and 9 different toppings. In how many ways can a pizza be made with the following ingredients?

a. 2 cheese and 4 toppings

b. 1 cheese and 3 toppings

Ex. 7 In a recent survey of 25 voters, 17 favor a new city regulation and 8 oppose. Find the probability that in a random sample of 6 respondents from this survey, exactly 2 favor the proposed regulation and 4 oppose it.

Ex. 8 A test consists of 25 questions and students are told to answer 20 of them. In how many different ways can they choose the 20 questions?

When reading a problem, you need to determine whether the problem involves permutations or combinations. In the following 5 questions, determine whether each situation involves a permutation or a combination.

Ex. 9

1. Four recipes were selected for publication and 302 were submitted.
2. Nine players are selected from a team of 15 to start the softball game.
3. Four out of 200 contestants were awarded prizes of $100, $75, $50, and $25.
4. A president and Vice-president are elected for a class of 210 students.
5. The batting order for the 9 starting players is announced.

Section 9-4 Using Addition with Probability

Objectives: Find the probabilities of mutually exclusive events.
Find the probabilities of inclusive events.

Events that cannot occur at the same time are called ________________________________.

Probability of *A or B*
Let *A* and *B* represent events in the same sample space.
If *A* and *B* are mutually exclusive events, then
***P*(*A or B*) = P(A) + *P*(*B*).**
If *A* or *B* are inclusive events, then
***P*(*A or B*) = *P*(*A*) + *P*(*B*) − *P*(*A and B*).**

The _________________ of event *A*, written _______, consists of all outcomes in the sample space the are not in *A*. For example, let *A* be the event in "favor." Then the complement A^c is the event "opposed" *or* "no opinion."

Probability of the Complement of *A*
Let *A* represent an event in the sample space.
$P(A)+P(A^c)=1 \quad P(A)=1-P(A^c) \quad P(A^c)=1-P(A)$

Use the given probability to find $P(E^c)$.

1. $P(E)=5/8$
2. $P(E)=0$
3. $P(E)=.528$
4. $P(E)=1$

Two number cubes are rolled. The table shows the possible outcomes. Use the table to state whether the events in each pair below are inclusive or mutually exclusive. Then find the probability of each pair of events.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

1. a sum of 6 or a sum of 10

2. a sum of 3 or a sum of 7

3. a sum of 9 or a sum of 5

4. a product of 20 or a sum of 6

5. a sum less then 10 or a sum greater than 8

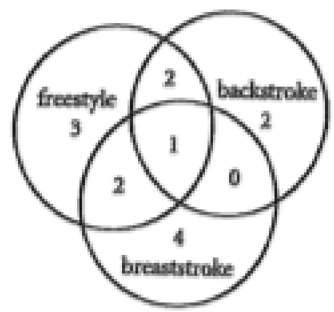
6. a sum of 3 or less, or double ones

7. a product of 5 or less, or sum of 6

8. an even sum or a sum of 7

9. an odd sum or a product greater than 25

10. a product greater than 20 or a product less than 15



A swim team with 25 members has 8 swimmers who swim freestyle, 5 swimmers who swim backstroke. Some swimmers participate in more than one event according to the Venn diagram. Find the probability of each event if a swimmer is selected at random.

11. swims freestyle

12. swims exactly 2 events

13. Swims breaststroke and backstroke

14. does not swim backstroke

15. swims freestyle and backstroke

16. does not swim freestyle, breaststroke, or backstroke

A number cube is rolled once, and the number on the top is recorded. Find the probability of each event.

1. 5 or 6
2. odd or even
3. Not even
4. 1 or less than 4

A card is drawn from a standard 52-card deck. Tell whether the events A and B inclusive or mutually exclusive. Then find P(A or B).

1. A: The card is red.
   B: The card is a 4.

____

2. A: The card is a face card.
   B: The card is a club.

____

3. A: The card is black.
   B: The card is red.

____

4. A: The card is heart or spade.
   B: The card is not a heart.

____

5. A: The card is less than 10.
   B: The card is a red face card.

____

6. A: The card is not a face card.
   B: The card is an ace.

____

7. A: The card is an ace of clubs.
   B: The card is red.

____

8. A: The card is red.
   B: The card is not a diamond or a heart.

____

Find the probability of each event. (honors)

1. 1 head or 2 tails appearing in 2 tosses of a coin.
2. 3 heads or 1 head appearing in 3 tosses of a coin.
3. At least 2 heads appear in 4 tosses of a coin.

Section 9-5 Independent Events

Objective: Find the probability of two or more independent events.

Two events are ________________________ if the occurrence or non-occurrence of one event has no effect on the likelihood of the occurrence of the other event. If one event does affect the occurrence of the other, the events are ___________________.

Probability of Independent Events
Events *A* and *B* are independent events if and only if $P(A and B) = P(A) \times P(B)$. Otherwise, *A* and *B* are dependent events.

Events A, B, C, and D are independent, and P(A) = 0.3, P(B) = 0.5, P(C) = 0.4, and P(D) = 0.1. Find each probability.

1. P(A and B) ______________________
2. P(C and B) ______________________
3. P(A and D) ______________________
4. P(B and D) ______________________

A spinner has 8 congruent areas where each area is exactly $\frac{1}{8}$ of the circle each numbered 1 though 8. Find the probability of each event in three spins of the spinner.

5. All three numbers are 3 or greater than 5.

~~6. All three numbers are odds.~~

A bag contains 6 red chips, 9 white chips, and 5 blue chips. A chip is selected and then replaced. Then a second chip is selected. Find probability of each event.

7. Both chips are white ______________________________

8. Neither chip is blue. ______________________________

9. The first chip is red and the second chip is white ______________________________

10. The first chip is blue and the second chip is not blue ______________________________

11. The first chip is not red and the second chip is not white______________________________

12. The first chip is red, white or blue and the second chip is red, white or blue ______________________________

Section 9-6 Dependent Events and Conditional Probability

Objective: Find conditional Probabilities.

The probability of event B, Given Event A has happened (or will happen) is called__________ ____.

Conditional Probability

The Conditional Probability of an event B, given event A denoted by $P(B|A)$, is given by $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$, where $P(A) \neq 0$.

A box contains 5 purple marbles, 3 green marbles, and 2 orange marbles. Two consecutive draws are made from the box without replacement of the first draw. Find the probability of each event.

1. Purple first, orange second _________________ 2. Green first, purple second _____________

3. green first, green second ___________________ 4. Orange first, green second _____________

5. orange first, purple second _________________ 6. Orange first, blue second ______________

7. purple first, purple second _________________ 8. Purple first, blue second ______________

Let A and B represent events.

9. Given P(A and B) = $\frac{1}{2}$ and P(A) = $\frac{2}{3}$, find P(B|A). ________________________________

10. Given P(A and B) = .12 and P(A) =0.2, find P(B|A). ________________________________

11. Given P(A) = $\frac{1}{4}$ and P(B|A) = $\frac{1}{3}$, find P(A and B). ________________________________

12. Given P(A) = 0.37 and P(B|A) =0.42, find P(A and B). ________________________________

13. Given P(B|A) = $\frac{2}{3}$ and P(A and B) = $\frac{1}{5}$, find P(A). ________________________________

14. Given P(B|A) = 0.63 and P(A and B) =0.27, find P(A). ________________________________

Is there a relationship between fruit consumption and amount of physical activity?

Fruit ↓/Exercise →	Low	Medium	High	**Total**
Low	69	206	294	**569**
Medium	25	126	170	**321**
High	14	111	169	**294**
Total	**108**	**443**	**633**	**1184**

Find the probability of a person:

a) P(High Fruit)

b) P(High Fruit Ç High Exercise)

c) P(High Fruit | High Exercise)

d) P(High Exercise | High Fruit)

Two number cubes are rolled and the first shows a 3. Find the probability of each event.

15. Both numbers are 3s.

____

16. A sum of 7.

____

17. The numbers are both odd.

____

18. A sum 2.

____

For one roll of a number cube, let A be the event “multiple of 2” and let B be the event “factor of 12.” Find each probability.

19. P(A) ________________________________

20. P(A and B) ________________________________

21. P(B|A) ________________________________

22. P(A|B) ________________________________