* Remember: $\log a$ is $\log_{10} a$
$\ln a$ is $\log_e a$

5-2 Rules of Logarithms

Name	Property	Examples
Zero Rule anything to 0 power = 0	$\log_a 1 =$ 0	$\log_5 1 =$ 0 $\ln 1 =$ 0
Identity Rule "anything" to power of 1 is "anything"	$\log_a a =$ 1	$\log_4 4 =$ 1 $\log 10 =$ 1
Inverse Properties if logs and exps have same base they undo each other	$\log_a a^r =$ r $b^{\log_b M} =$ M	$\log_4 4^3 =$ 3 $\ln e^{-0.5} =$ -0.5 $5^{\log_5 20} =$ 20 $e^{\ln 24} =$ 24
Product Rule recall: $x^2 \cdot x^4 = x^{2+4} = x^6$	$\log_b MN =$ $\log_b M + \log_b N$	$\log_2 5 * 3 = \log_2 5 + \log_2 3$ $\log 5w = \log 5 + \log w$ $\ln 6z = \ln 6 + \ln z$
Quotient Rule recall: $\frac{x^6}{x^2} = x^{6-2} = x^4$	$\log_b \frac{M}{N} =$ $\log_b M - \log_b N$	$\log_7 \frac{9}{x} = \log_7 9 - \log_7 x$ $\ln \frac{p}{3} = \ln p - \ln 3$
Power Rule $\log_2 x^2$	$\log_b M^r = r \log_b M$	$\log_8 3^5 = 5 \log_8 3$ $\log_b 5^2 = 2 \log_b 5$

$= \log_2 x \cdot x$

$= \log_2 x + \log_2 x$

$= 2 \log_2 x$ so $\log_2 x^2 = 2 \log_2 x$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

Rules of Logarithms Examples:

Expand using the Rules of Logarithms.

$\log_2 x^2 y^2$

$\log_2 x^2 + \log_2 y^2$

$2\log_2 x + 2\log_2 y$

$\log \frac{x^3}{\sqrt{y}}$

$\log x^3 - \log \sqrt{y}$

$3\log x - \frac{1}{2}\log y$

$\log_3 \frac{ab^2}{c^3}$

$\log_3 ab^2 - \log_3 c^3$

$\log_3 a + \log_3 b^2 - \log_3 c^3$

$\log_3 a + 2\log_3 b - 3\log_3 c$

Using the Rules of Logarithms, write each expression as a single logarithm (condense), then simplify if possible.

$\log_6 3 + \log_6 12$

$\log_6 3 \cdot 12$

$\log_6 36 = 2$

$3\log_5 x - \frac{1}{3}\log_5 y$

$\log_5 x^3 - \log_5 \sqrt[3]{y}$

$\log_5 \frac{x^3}{\sqrt[3]{y}}$

$\ln 2 + 3\ln a - 4\ln b$

$\ln 2 + \ln a^3 - \ln b^4$

$\ln 2a^3 - \ln b^4$

$\ln \frac{2a^3}{b^4}$

$2\log_4 x + (\log_4 3y - 4\log_4 z)$

$\log_4 x^2 + (\log_4 3y - \log_4 z^4)$

$\log_4 x^2 + \log_4 \frac{3y}{z^4}$

$\log_4 \frac{3x^2y}{z^4}$

$\log_6 6 + \frac{1}{2}\log_6 36$

$= \log_6 6 + \log_6 36^{\frac{1}{2}}$

$= \log_6 6 + \log_6 \sqrt{36}$

$= \log_6 6 + \log_6 6$

$= \log_6 36$

$= 2$

$\ln e^{10} - \ln 4$

$\ln \frac{e^{10}}{4}$