

2.5 Complex Zeros

- Objectives:
- 1) I can find complex zeros of a polynomial.
 - 2) Given the complex factors or zeros, I can write a polynomial in standard form.
 - 3) I can determine how many complex, real, and total zeros of a polynomial.

Complex Numbers $a \pm bi$

$a + bi$ $a - bi$ $\frac{2}{2} = 1$ $\tilde{z} = 1$ $-1 + 2i$
 $x^2 + 8 = -1$ $x^2 + 2x + 4 = 0$ $-1 - 2i$
 $\sqrt{x^2 + 9}$ $x^2 + 2x = -4$
 $x = 3i, -3i$ $x^2 + 2x + 1 = -4 + 1$
 $\sqrt{(x+1)^2} = \sqrt{-3}$
 $a + bi$ $x + 1 = \pm \sqrt{3} i$
 $x = -1 \pm \sqrt{3} i$

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Write the polynomials in standard form.

$$\begin{aligned} f(x) &= (x-2i)(x+2i) \\ &= x^2 + 2ix - 2ix - 4i^2 \\ f(x) &= x^2 + 4 \end{aligned}$$

$$\begin{aligned} f(x) &= (x-5)(x-i\sqrt{2})(x+i\sqrt{2}) \\ &= (x-5)(x^2 + 2) \\ &= x^3 + 2x - 5x^2 - 10 \end{aligned}$$

$$f(x) = (x-3)(x-3)(x-i)(x+i)$$

Write the polynomials of minimum degree from the given zeros. $x=1, x=1, x=1+2i, x=1-2i$

$x=1$ $f(x) = (x-1)^2 (x-1-2i)(x-1+2i)$

$x-1 = 0$

$$(x-1-2i)$$

$$(x-1+2i)$$

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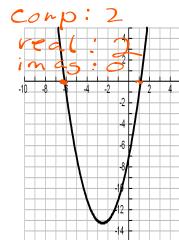
Write the polynomial of minimum degree from the zeros and multiplicities given:

$\begin{matrix} x = -1 \\ x+1 = 0 \end{matrix}$
-1 (with multiplicity 3) and 3 (with multiplicity 1)

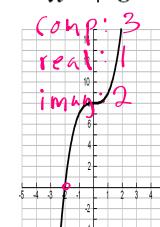
$$f(x) = (x+1)^3 (x-3)$$

How many **real zeros** and how many **imaginary zeros** are there in each function?

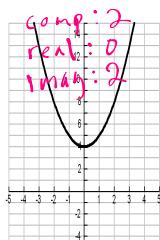
$$x^2 + 5x - 7$$



$$x^3 + 8$$



$$x^2 + 4$$



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Finding complex zeros

$$f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$$

1st) Rational Roots Theorem or Calculator

$$x = \pm \frac{1, 2, 4, 8}{1} = \pm 1, 2, 4, 8$$

$$\begin{array}{r} \boxed{1} & -3 & -5 & 5 & -6 & 8 \\ & 1 & -2 & -7 & -2 & -8 \\ \hline & 1 & -2 & -7 & -2 & -8 & |0 \\ & & x^4 - 2x^3 - 7x^2 - 2x - 8 \end{array}$$

$$\begin{array}{r} \boxed{-2} & 1 & -2 & -7 & -2 & -8 \\ & -2 & 8 & -2 & 8 \\ \hline & 1 & -4 & 1 & -4 & |0 \\ & & x^3 - 4x^2 + x - 4 \end{array}$$

$$\begin{array}{r} \boxed{4} & 1 & -4 & 1 & -4 \\ & 4 & 0 & 4 \\ \hline & 1 & 0 & 1 & |0 \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = i, -i$$

Given the **complex zero** $1-2i, 1+2i$ of the **function** $f(x) = 4x^4 + 17x^2 + 14x + 65$.

Find the remaining zeros.

$$\begin{aligned} x &= 1-2i & x &= 1+2i \\ (x-1+2i)(x-1-2i) & & & \\ x^2 - x - 2ix - x + 1 + 2i & & + 2ix - 2i - 4i^2 \\ \downarrow & & & \\ x^2 - 2x + 5 & & & \\ & & & \cancel{4x^4 + 0x^3 + 17x^2 + 14x + 65} \\ & & & - \cancel{4x^4 - 8x^3 + 20x^2} \\ & & & \cancel{8x^3 - 3x^2 + 14x + 65} \\ & & & - \cancel{8x^3 - 16x^2 + 40x} \end{aligned}$$

$$\begin{aligned} \frac{a}{4x^2} &= b & c &= \\ 4x^2 + 8x + 13 & & & - \frac{8x^3 - 3x^2 + 14x + 65}{8x^3 - 16x^2 + 40x} \\ & & & \cancel{13x^2 - 76x + 65} \\ x &= -8 \pm \sqrt{\frac{8^2 - 4(4)(13)}{64 - 208}} & & \cancel{17x^2 - 26x + 65} \end{aligned}$$

$$= -8 \pm \frac{12i}{8}$$

$$= -1 \pm \frac{3}{2}i$$

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Linear Factorization

$$f(x) = 2x^3 - 6x^2 + 4x$$

$$2x(x-1)(x-2)$$



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