12-3 Step Functions

Objectives:

I can write and graph step function problem situations.
I can analyze the graphs of step functions.
I can use a calculator to graph a step function.

Aug 26-9:30 AM

In 2004, Georgia had 6 income tax brackets. The tax rate on every dollar of income was:

- 1% for incomes more than \$0 and up to and including \$750
- 2% for incomes more than \$750 and up to and including \$2250
- 3% for incomes more than \$2250 and up to and including \$3750
- 4% for incomes more than \$3750 and up to and including \$5250
- 5% for incomes more than \$5250 and up to and including \$7000
- 6% for incomes more than \$7000

1. Write a piecewise function f(x) for the tax paid in Georgia for income x.

Aug 26-9:32 AM

2. Calculate the amount of tax paid with an income of:

a. \$750

b. \$751

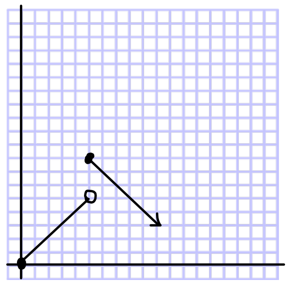
Aug 26-9:35 AM

Let's consider the linear piecewise function, f(x), and how to create a graph.

$$f(x) = \begin{cases} x, 0 \le x < 5 \\ -x+13, x \ge 5 \end{cases}$$

Graph each interval of the function according to its domain.

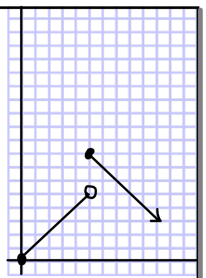
Use the domain of each interval to determine if endpoints will be included



Aug 26-9:36 AM

3. Analyze the linear piecewise function in the worked example.

a. How does this function differ from other linear piecewise functions you have seen so far?

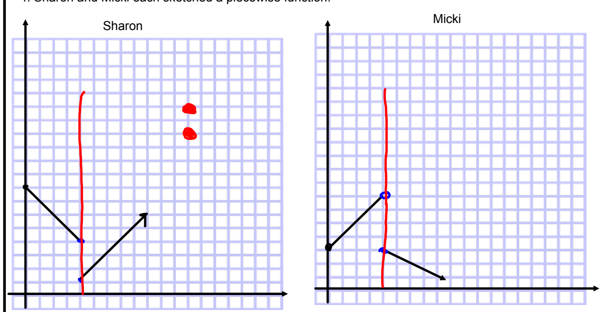


b. How do you know whether an endpoint should be included in the graph or represented as an open circle?

c. Notice that the function is not continuous. At what x-value is there a break in the graph? Why do you think that break occurs?

Aug 26-9:43 AM

4. Sharon and Micki each sketched a piecewise function.



Sharon is unsure why her function is incorrect, but Micki's is correct. Prove why Sharon's graph is not a function.

Aug 26-9:45 AM

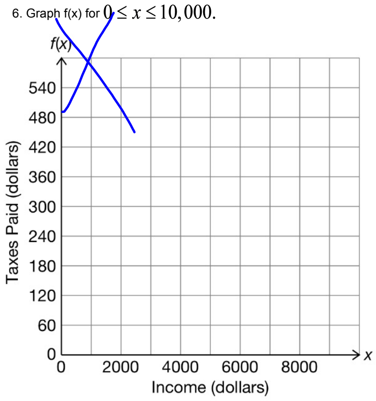
5. Analyze the function you wrote to represent the tax brackets in Georgia from Question 1.

   a. Would the lines that represent each piece of the function on the graph be connected or contain breaks? Explain.

   b. Describe the endpoints of the line representing each interval. Explain.

Aug 26-9:51 AM

6. Graph f(x) for $0 \le x \le 10,000.$



Aug 26-9:52 AM

7. Describe the rate of change when:

   a. $0 \le x \le 750$

   b. $750 < x \le 1000$

Aug 26-10:01 AM

8. Calculate the amount of tax paid on an income of:

   a. $2250

   b. $2251

   c. $7000

   d. $7001

9. Describe the method you used to calculate the amount of tax for year income.

Aug 26-10:03 AM

Problem 2: Taxi Fares

In 2006, the rate for a taxi ride in Macon, Georgia, was $1.20 for the first mile or part of a mile, and $1.20 for each additional mile or part of a mile.

1. Define a piecewise function, g(x), for the cost of a taxi ride up to 5 miles.

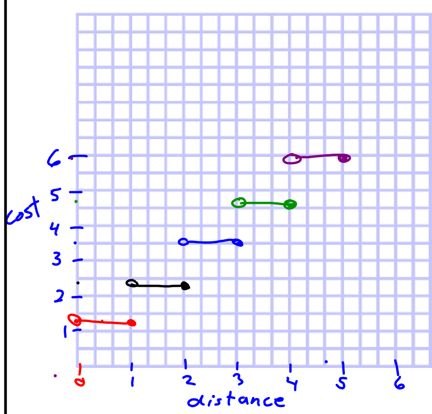
$$g(x) = \begin{cases} 1.20, & 0 < x \le 1 \\ 2.40, & 1 < x \le 2 \\ 3.60, & 2 < x \le 3 \\ 4.80, & 3 < x \le 4 \\ 6.00, & 4 < x \le 5 \end{cases}$$

2. What is the slope of each interval? Explain your reasoning.

$m = 0$

Aug 26-10:04 AM

3. Graph g(x) for x < 5 miles.



Aug 26-10:06 AM

4. Describe the graph of the function as either increasing or decreasing.

You have just graphed a *step function.* A **step function** is a piecewise function whose pieces are disjoint constant functions.

5. Why do you think this function is called a step function?

Aug 26-10:07 AM

6. Use a graphing calculator to graph g(x).

7. Can you determine by viewing your graph on a graphing calculator whether an endpointe is included or not in the graph?

Aug 26-10:08 AM

Problem 3 Special Step Functions

The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**, $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

1. Evaluate each using the greatest integer function.

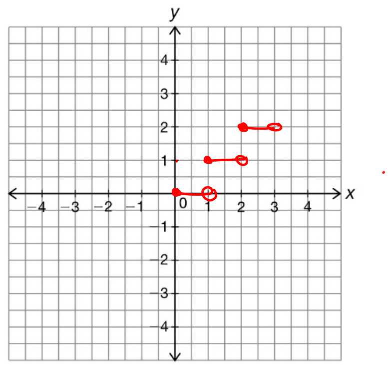
a. $\lfloor 2 \rfloor =$ 2 **b.** $\lfloor 0.17 \rfloor =$ 0

c. $\lfloor 2.34 \rfloor =$ 2 **d.** $\lfloor -1.2 \rfloor =$ -2

e. $\lfloor 2.99999 \rfloor =$ 2 **f.** $\lfloor -0.2 \rfloor =$ -1

Aug 26-11:41 AM

2. Graph $G(x) = \lfloor x \rfloor$.



Aug 26-11:49 AM

The *least integer function* is another special kind of step function. The **least integer function** $L(x) = \lceil x \rceil$ also known as the **ceiling function,** is defined as the least integer greater than or equal to x.

3. Calculate each:

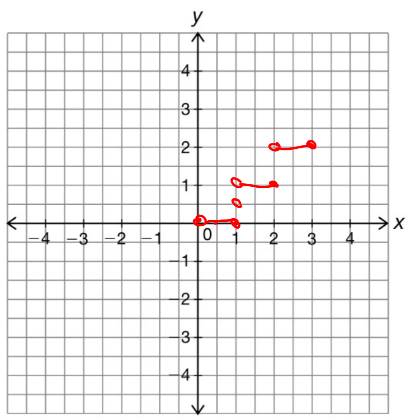
a. $\lceil 2 \rceil =$ 2 **b.** $\lceil 0.17 \rceil =$ __

c. $\lceil 2.34 \rceil =$ 3 **d.** $\lceil -1.2 \rceil =$ -1

e. $\lceil 2.99999 \rceil =$ __ **f.** $\lceil -0.2 \rceil =$ 0

Aug 26-11:50 AM

4. Graph $L(x) = \lceil x \rceil$.



Aug 26-11:52 AM

5. Compare the graphs you created for the greatest integer function and the least integer function. What do you notice?

6. Use your graphing calculator to graph $G(x) = \lfloor x \rfloor$
7. Use your graphing calculator to graph $L(x) = \lceil x \rceil$
8. Compare the graphs and the equations for the least integer function and the greatest integer function. What do you notice?

9. While the calculator provides a reasonable representation of the graph, why might it not be the best representation to use?

Aug 26-11:52 AM